

Short Bunch Production Near Transition

Microwave Instability near Transition

K. Y. Ng

June 30, 1999

For the muon collides proton driver, bunching to  $\sigma \lesssim 2\text{ns}$  is desirable.

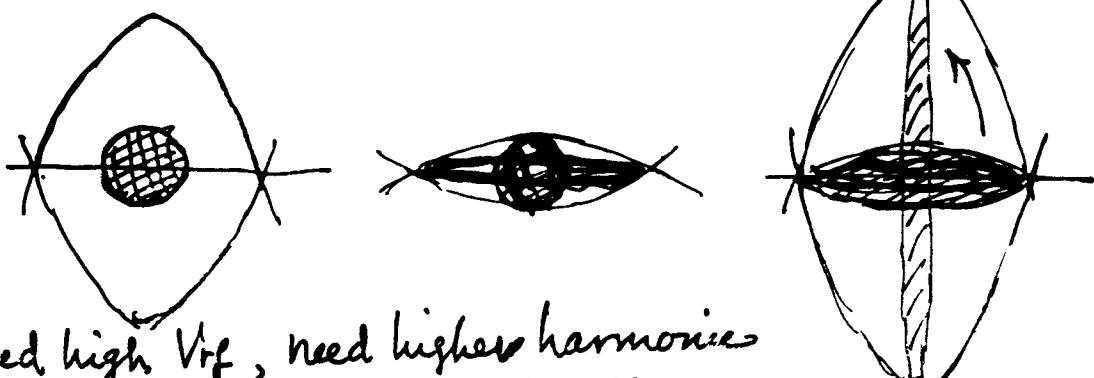
Reasons.

1. Only piece of information transmitted to the  $\pi'$ s and  $\mu'$ s. Thus less cooling will be required if the proton bunch hitting the target is as narrow as possible.
2. polarized  $\mu$  states are easier to separate.

How to make short & intense bunches at low energy?

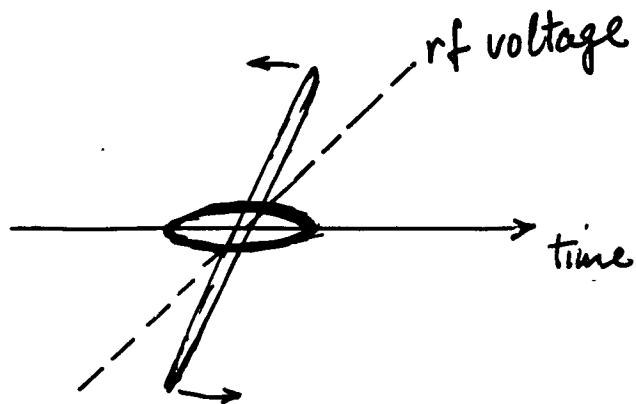
1. Adiabatic spreading of bunch followed by raising  $V_f$  a bunch rotation.

There may be microwave instability during the slow spreading  
Can do this fast by snapping  $V_f$



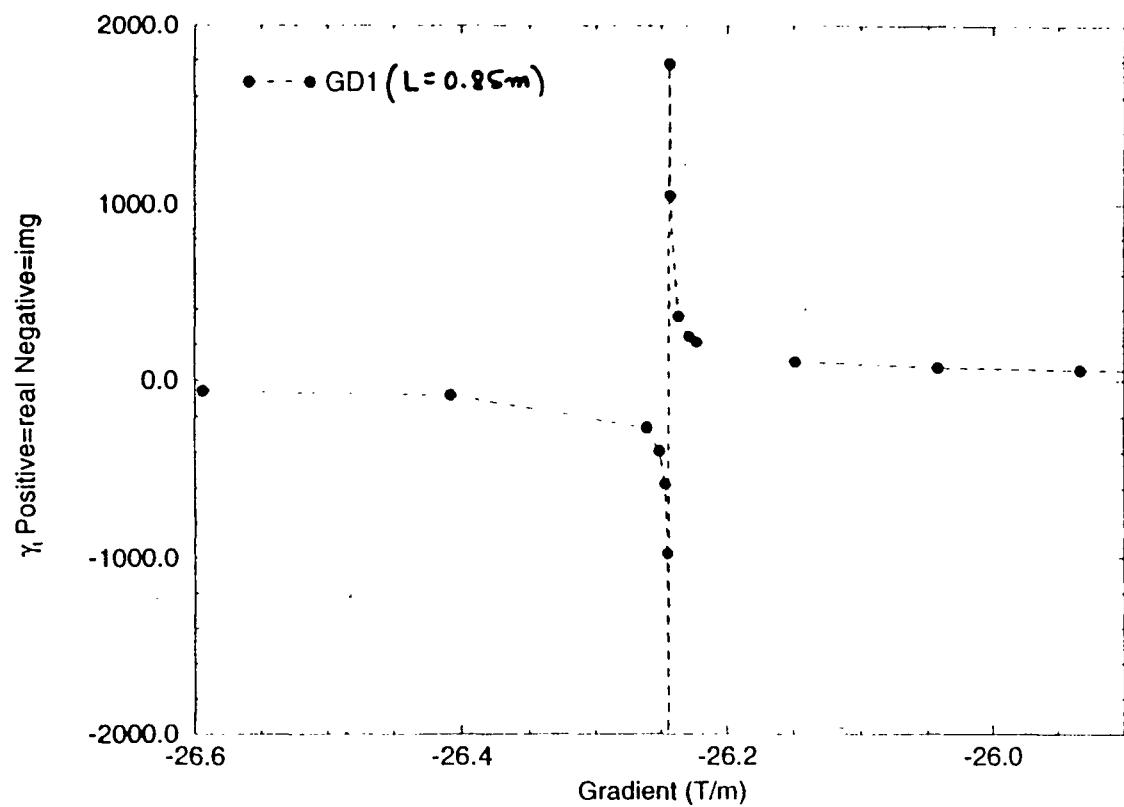
need high  $V_f$ , need higher harmonics  
coincide to cancel nonlinearity.

2. Rebunching at higher rf frequency  
need high frequency & high voltage cavities
3. Energy accumulation near transition, followed by a partial rf rotation. (idea of Jim Norem)



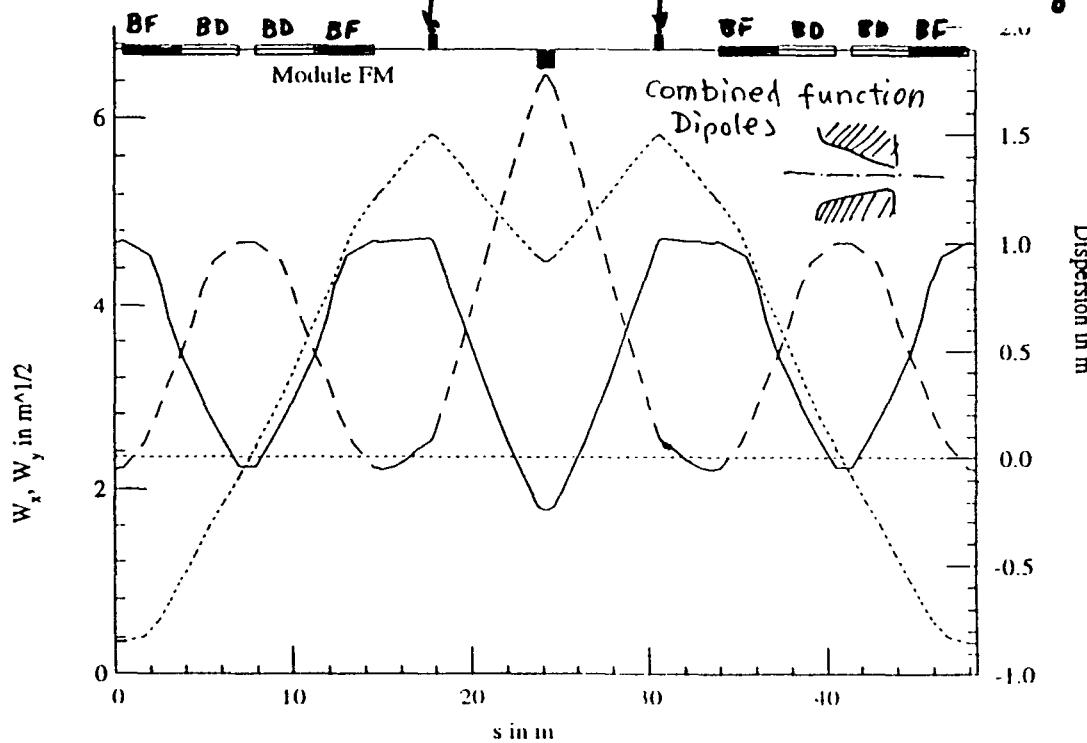
However, we do not need to operate the ring near transition all the time. With FMC lattice, we can move close to transition only near extraction when the production of short bunches is required.

The merit of this method is that no additional hardware, such as harmonic cavities, are required.

$\gamma_i$  Dependence on QUAD STRENGTH

MC lattices can move  $\gamma_i$  easily and, presumably, quickly.

vary the gradient of these 2 quads will change  $\gamma_i$



Dispersion max/min: 1.49327/-0.85000m,

$\beta_x$  max/min: 22.22/ 3.12487m,  $v_x$ : 19.90411,  $\xi_x$ : -24.32,

$\beta_y$  max/min: 41.86/ 4.88857m,  $v_y$ : 19.07570,  $\xi_y$ : -28.04,

$\gamma_i$ : (-60.23, 0.00)

Module length: 1163.8806m

Total bend angle: 6.28318548 rad

Trbojevic '96

The experiment was performed at BNL AGS in 1997.

Expt. E 932

## Bunching Near Transition in the AGS

C. Ankenbrandt, K-Y. Ng, J. Norem,  
M. Popovic, Z. Qian  
FNAL

L. A. Ahrens, M. Brennan, V. Mane,  
T. Roser, D. Trbojevic, W. van Asselt  
BNL

Phys. Rev. ST Accel Beams  
030101 (1998)

BNL, 6/99

## Operating Mode of the AGS

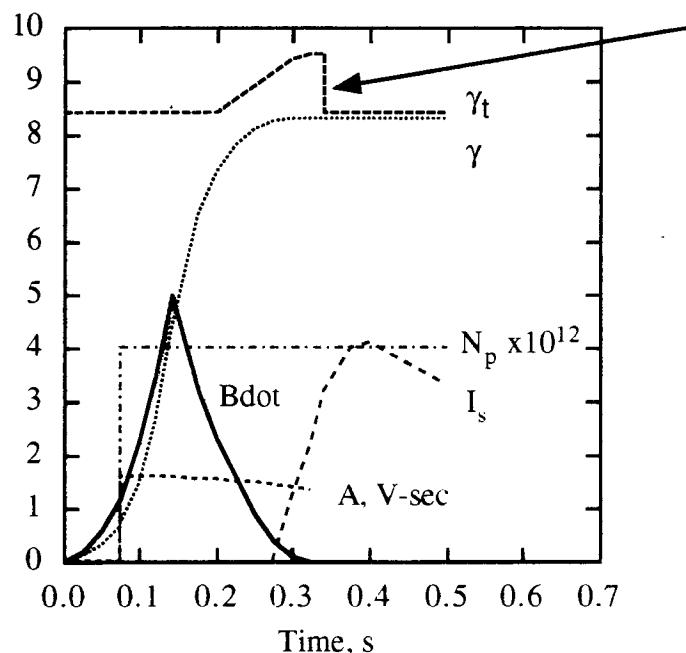
$$n_p \sim 4 \times 10^{12} / \text{bunch}$$

$$-0.05 < \gamma_t - \gamma < 0.05$$

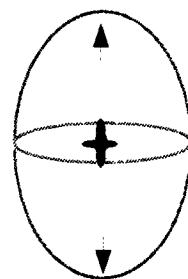
$$E \sim 7 \text{ GeV}$$

$$A \sim 1.5 \text{ eV/s}$$

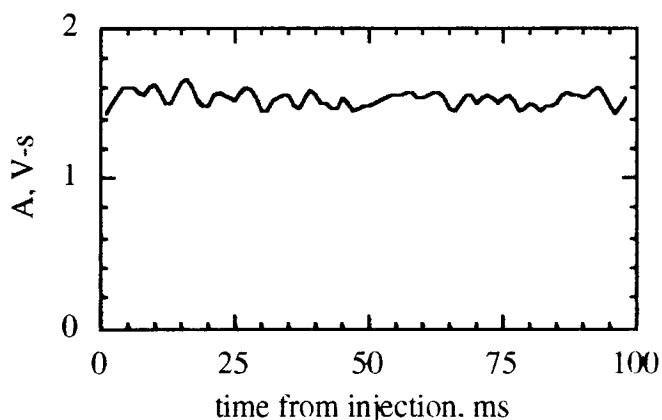
Machine parameters:



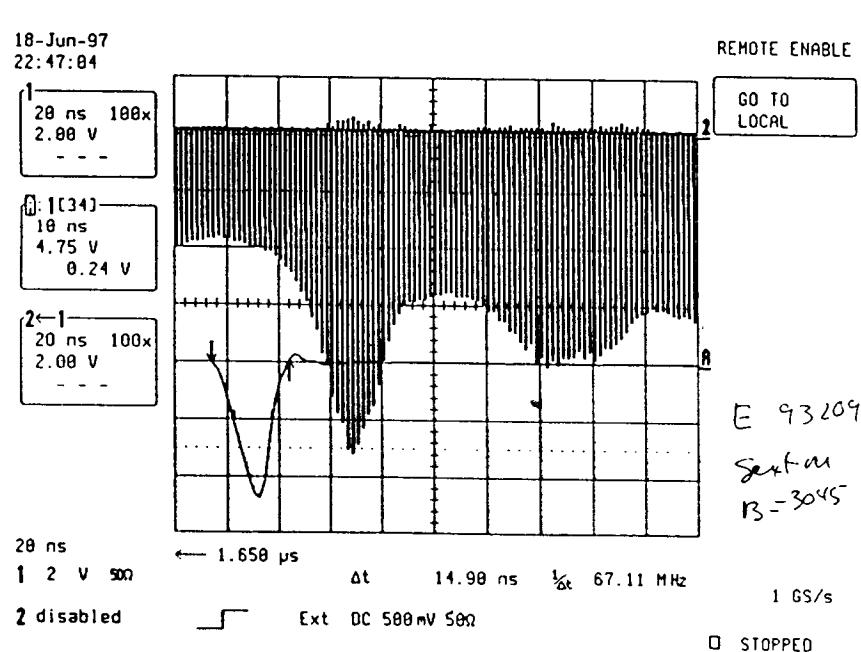
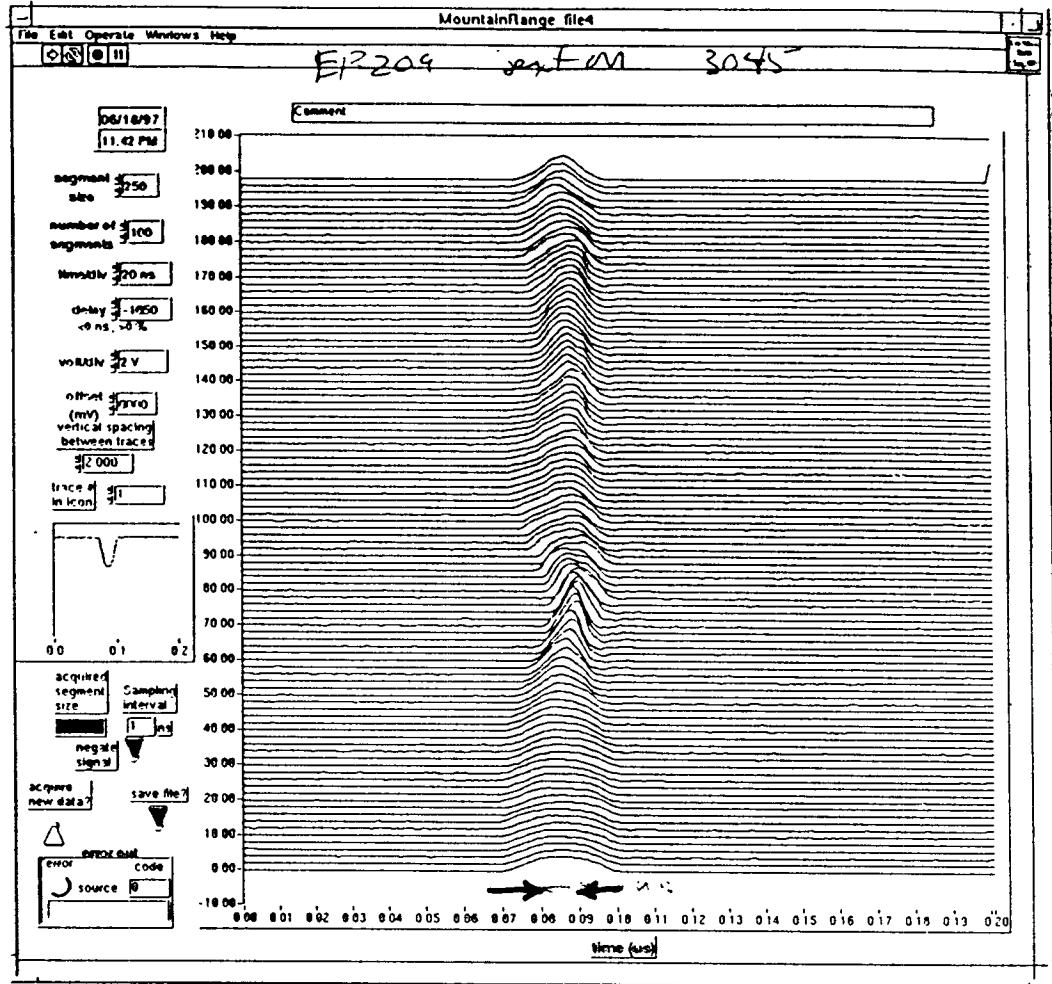
Suddenly makes bunch very tall.

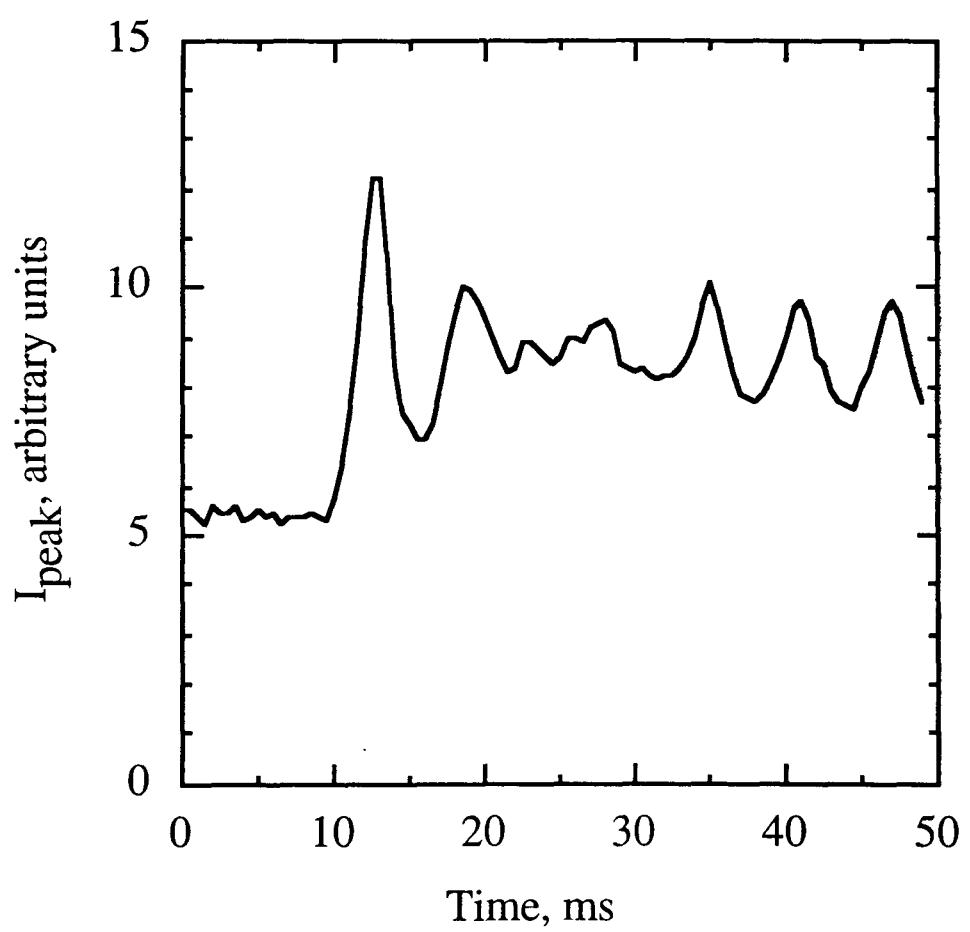


Bunch area measurements:



1. Beam cannot be too close to  $\gamma_t$ , otherwise the final rotation will take very long.
2. Beam cannot be too far from  $\gamma_t$ , otherwise the bunch will not be tall enough to produce a tall but narrow bunch.





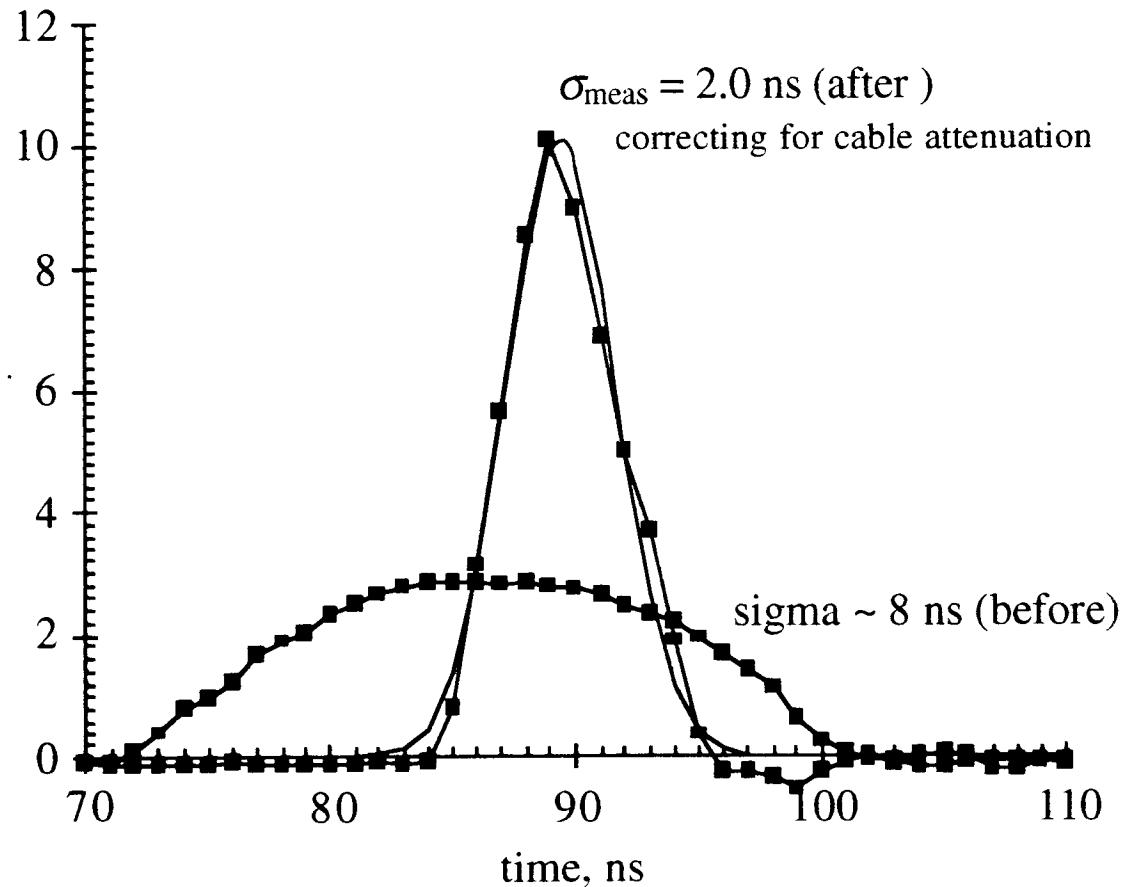
## Bunching has been Demonstrated at the AGS

Bunching of the proton beam to  $\sigma \sim 2$  ns is desirable because:

- Less cooling is required if the initial proton / pion / muon bunch is as small as possible.
- Polarization states are easier to separate by bunch rotation if the bunch is short.

The bunch rotation was done by dropping  $\gamma_t$  to the beam  $\gamma$ .

- $\epsilon_L$ , AGS  $\sim \epsilon_\mu$  collider driver, charge  $\sim 1/10$  required
- 2 ns at 3 MHz  $\Rightarrow \sim 1$  ns at 7 MHz
- Other options and better tuning are possible
- short bunches were stable



## Comments

### 1. Nonlinear dispersion

$$\gamma(\delta) = \gamma_0 + \frac{1}{\gamma^2} (\alpha_1 + \frac{3\beta}{2}) \delta + O(\delta^2)$$

$\downarrow$

$$\frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}$$

So the bunch will shear nonlinearly

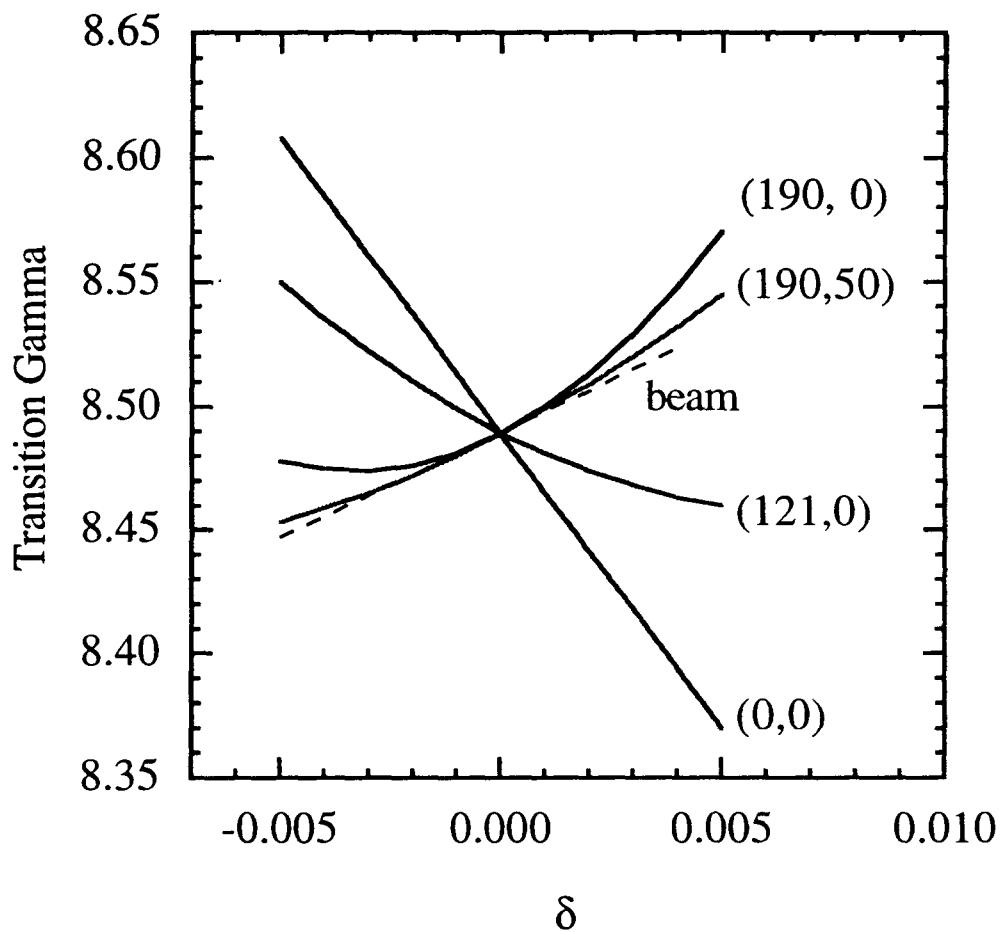
thus the bunch will become wider (can correct  $\alpha_1$  to  $-\frac{3}{2}$   
by sextupoles)

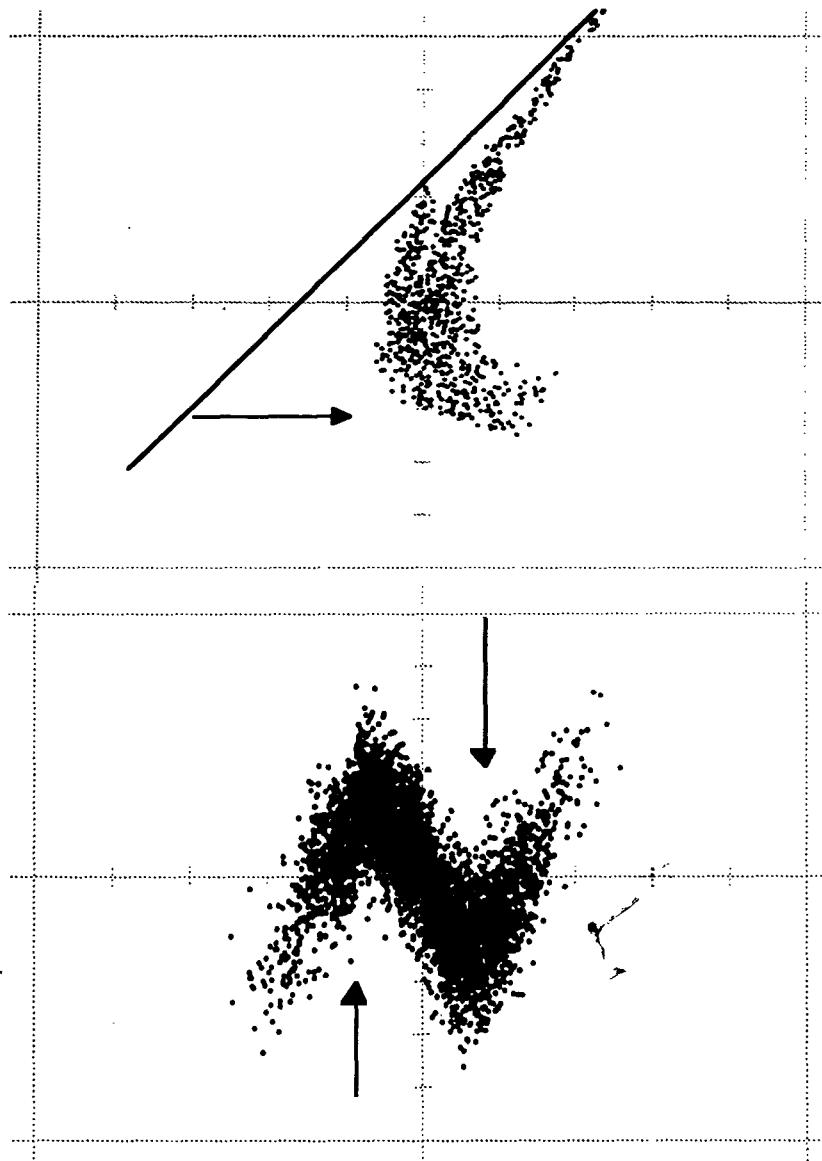
### 2. Space charge effect $\Rightarrow$ bunch shape becomes



- if too close to  $\gamma_t$
- can compensate by rf.

### 3. No beam instability observed





The plots span a phase space area of  $\pm 100$  MeV and  $\pm 40$  ns.

## Microwave Instability near transition

$$\text{Since } \gamma(\delta) = \gamma_0 + \underbrace{\frac{1}{\gamma_2} (\alpha_1 + \frac{3\beta}{2}) \delta}_{\gamma_1} + O(\delta^2)$$

The  $\gamma_1$  term must be included for meaningful discussions.

Alex Bogacz did that in PAC'91, but with  $\gamma_0=0$

result for Gaussian distribution in energy (coasting beam)

$$\text{growth rate: } \frac{1}{\tau_n} = -2\alpha_1 n \omega_0 \left( \frac{\sigma_E}{E} \right)^2 \tan^{-1} \frac{\text{Im } Z_n}{\text{Re } Z_n}$$

$\text{Im } Z_n > 0 \Rightarrow \text{capacitive}$

$\therefore \text{assuming } \alpha_1 > 0 \quad \frac{\text{Im } Z_n}{\text{Re } Z_n} > 0 \Rightarrow \text{stable}$

$\gamma_0=0 \quad \alpha_1 > 0 \Rightarrow \text{above transition}$

But stable above transition & capacitive impedance  
is something strange.

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Jim Holt & Pat Colestock studied the same problem  
with  $\gamma_0 \neq 0$  but including  $\gamma_1$  PAC'95

Also with coasting beam & Gaussian energy distribution,  
they get dispersion relation in terms of complex error fun.

Their conclusion:

below transition: No unstable region in  $Re^2 - Im^2$  plan  
above transition: there are both stable &  
unstable region

They had simulations to support their claim  
However, they did not specify the values of  $\gamma_0$  &  $\gamma_1$ ,  
and others in their plots & simulations

It is hard for me to understand the situation below  
transition. According to their claim, there is no  
microwave instability at all below transition.

Notice that if  $|\gamma_0|$  is not too small,  $\gamma_1$  is  
irrelevant.

At least my own simulations show different  
results.

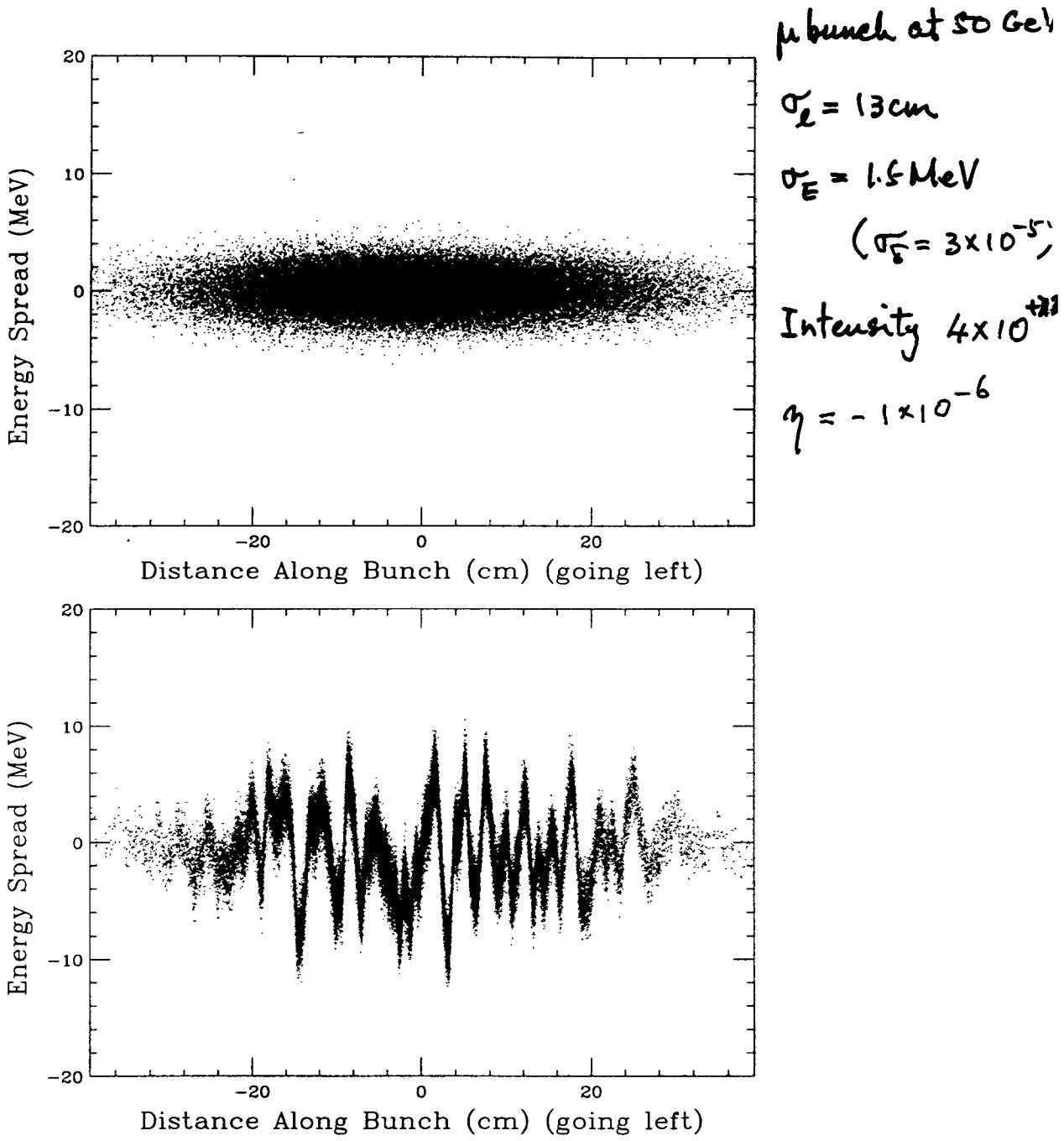


FIG. 5. Simulation of the 13-cm bunch of  $4 \times 10^{12}$  muons subject to a broad-band impedance with quality factor  $Q = 1$  and  $Z_{||}/n = 0.5 \Omega$  at the resonant angular frequency  $\omega_r = 50$  GHz. The half-triangular bin width is 15 ps (0.45 cm) and  $2 \times 10^6$  macro-particles are used. Top plot shows initial distribution with  $\sigma_E = 1.5$  MeV and  $\sigma_\ell = 13$  cm. Lower plot shows distribution after 1000 turns with compensating rf's initially at  $\omega_1/(2\pi) = 0.3854$  GHz and  $\omega_2/(2\pi) = 0.7966$  GHz, voltages  $V_1 = 65.40$  kV and  $V_2 = 24.74$  kV, and phases  $\varphi_1 = 177.20^\circ$  and  $\varphi_2 = 174.28^\circ$ . The rf voltages decrease according to the decay of the muons.

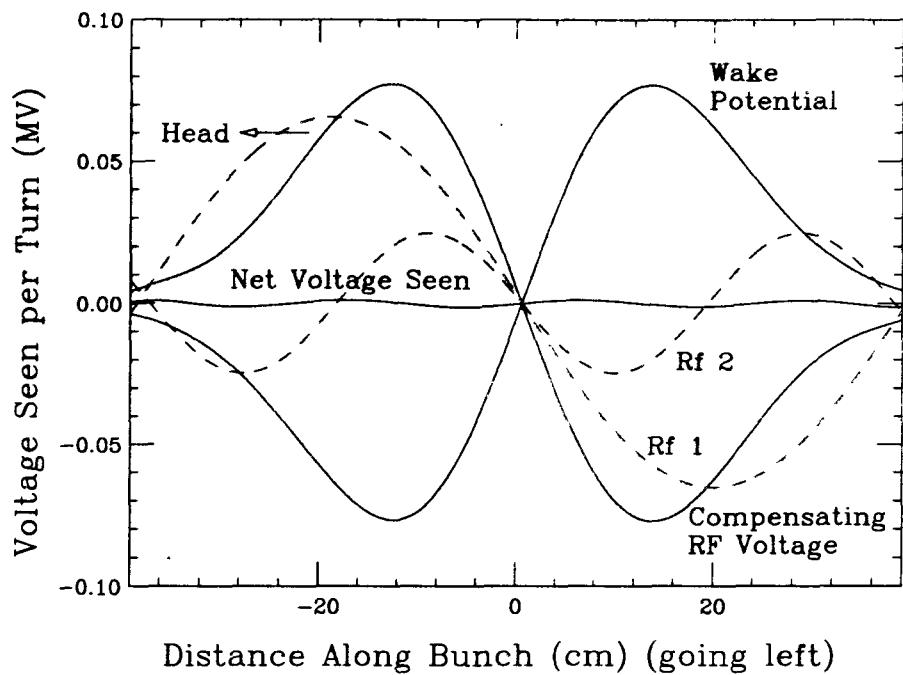


FIG. 4. Wake potential, compensating rf voltages, and net voltage seen by particles in the 13-cm bunch at injection. The compensating rf is the sum of two rf's represented by red and magenta dashes.

$\mu$  bunch at 50 GeV,  $\sigma_x = 13\text{cm}$ .  $\sigma_y = 1.5\text{ MeV}$  ( $\sigma_z = 3 \times 10^{-5}$ )

$$\text{Broad band impedance} \quad Q = 1 \quad \frac{Z}{n} = 0.5 \Omega$$

$$\gamma = 10^{-6}$$

$$f_r \sim \frac{\sigma_0}{2\pi} = 7.96 \text{ GHz}$$

$$\text{Keil Schnell limit: } \frac{Z}{n} \sim 10^{-7} \Omega$$

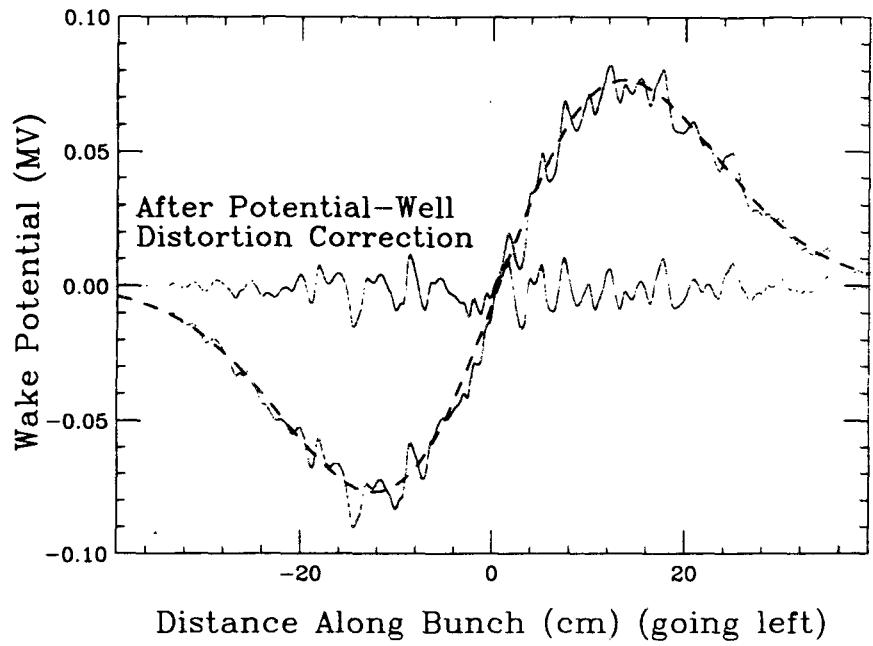


FIG. 6. Wake potential seen by the simulated bunch is shown by the red curve, which differs slightly from the wake potential of an ideal smooth Gaussian bunch shown in blue dashes. The difference shown by the magenta solid curve represents the random fluctuation of the finite number of macro-particles.

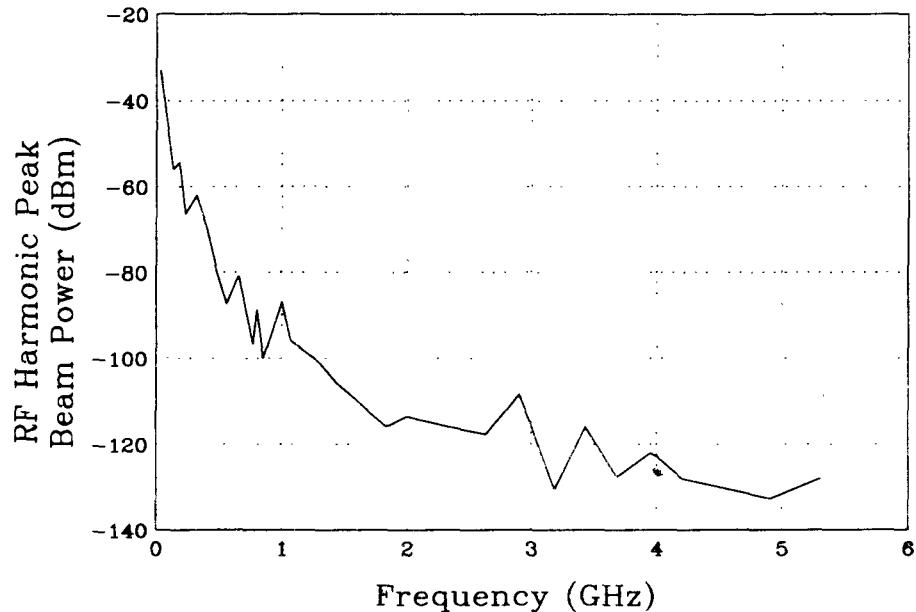
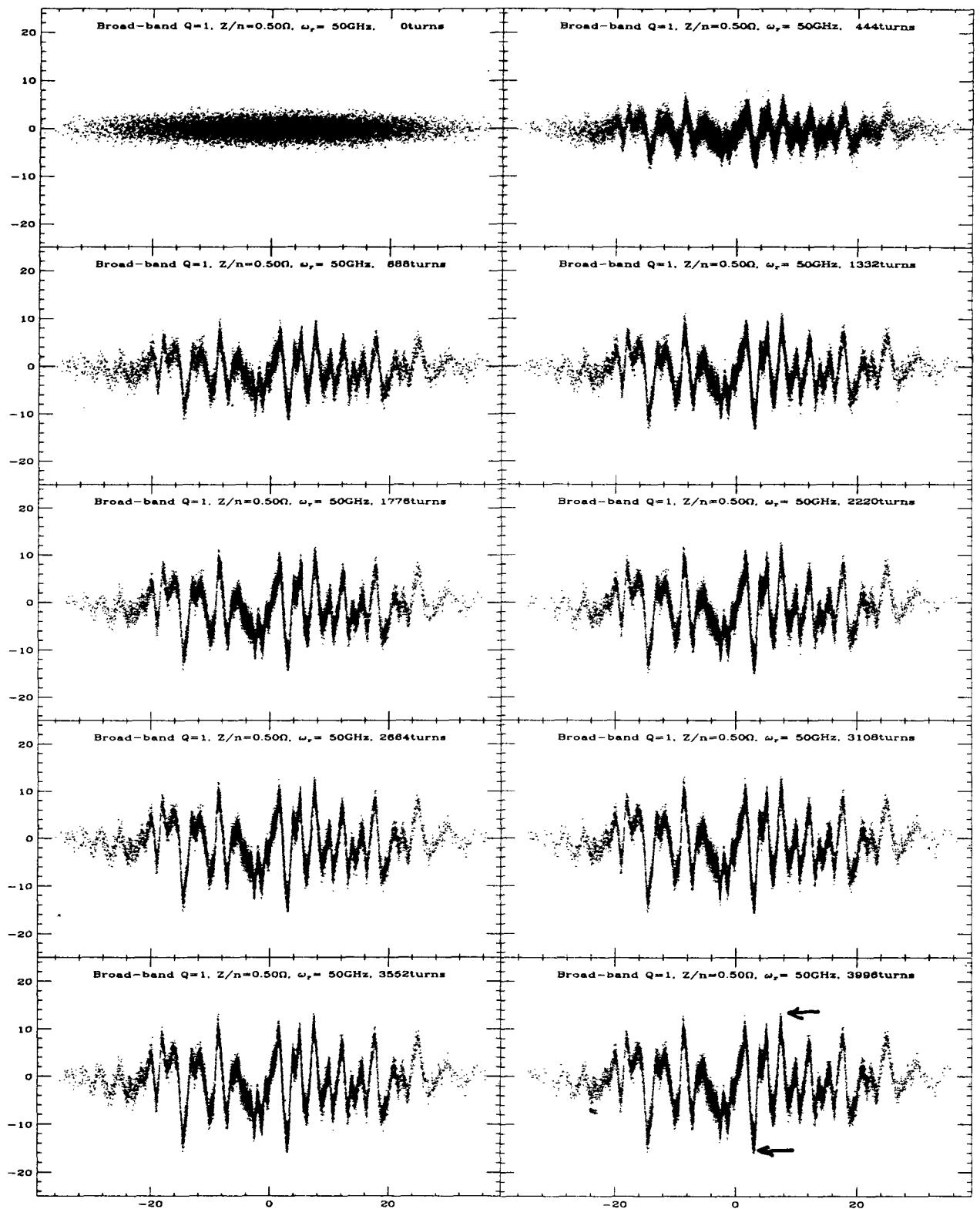
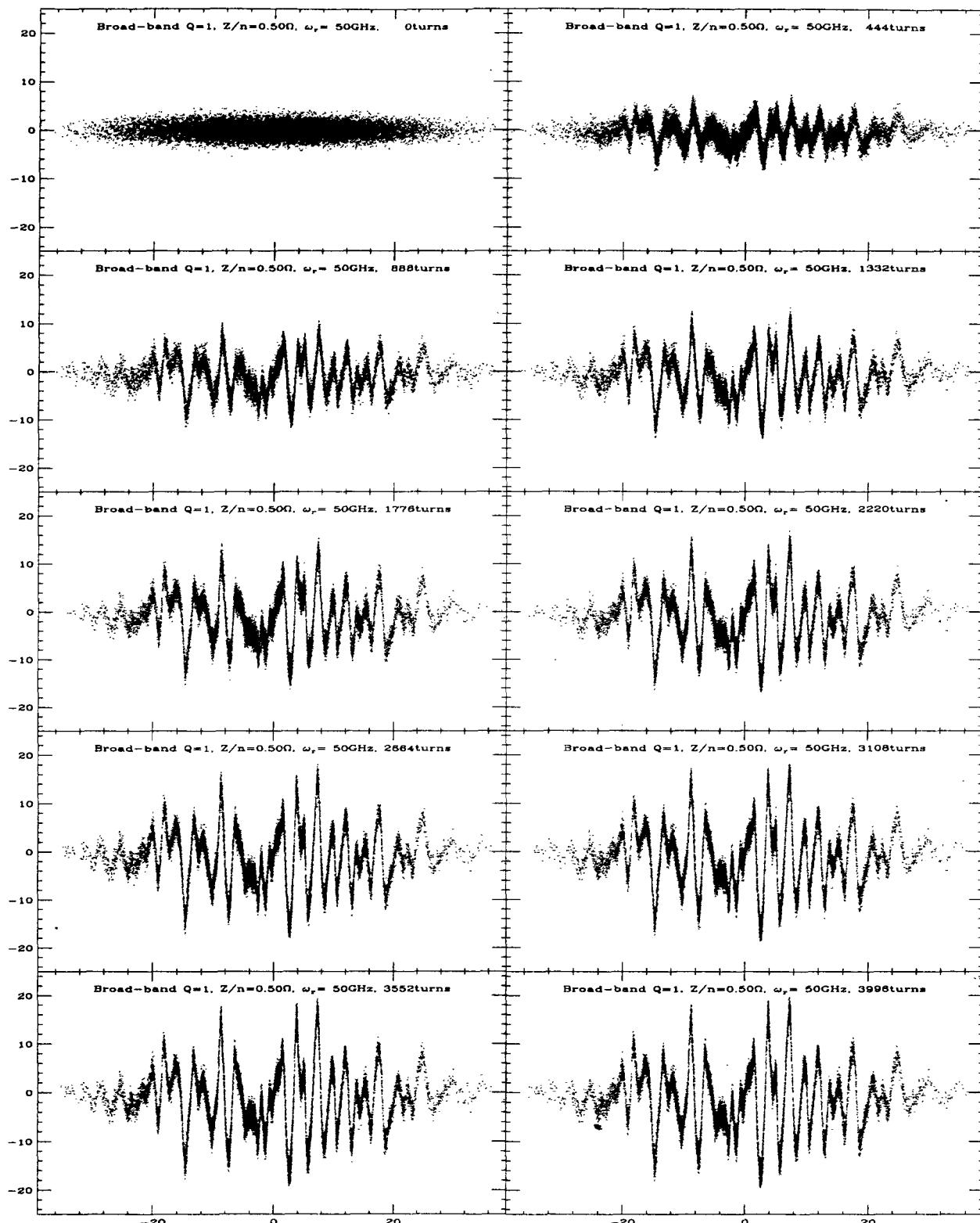


FIG. 7. Beam current power spectrum of the Tevatron, measured at each harmonic of the RF frequency.



Energy offset in MeV vs distance in cm,  $E = 50$  GeV with  $\sigma_E = 1.5$  MeV,  $\sigma_t = 13$  cm.  $\eta = -1 \times 10^{-10}$ ,  $4 \times 10^{12}$  particles ( $2 \times 10^6$  macro), half-triangle width 15 ps. Compensating rf's: 65.341, 24.645 kV,  $\omega = 2421.83773$ , 5005.13131 MHz,  $\phi = 178.60214^\circ, 177.15095^\circ$ . Decay of muons has been included, 4000 turns, File: 50ghz-5-13-decay-eta-10-4000turns-2e6.

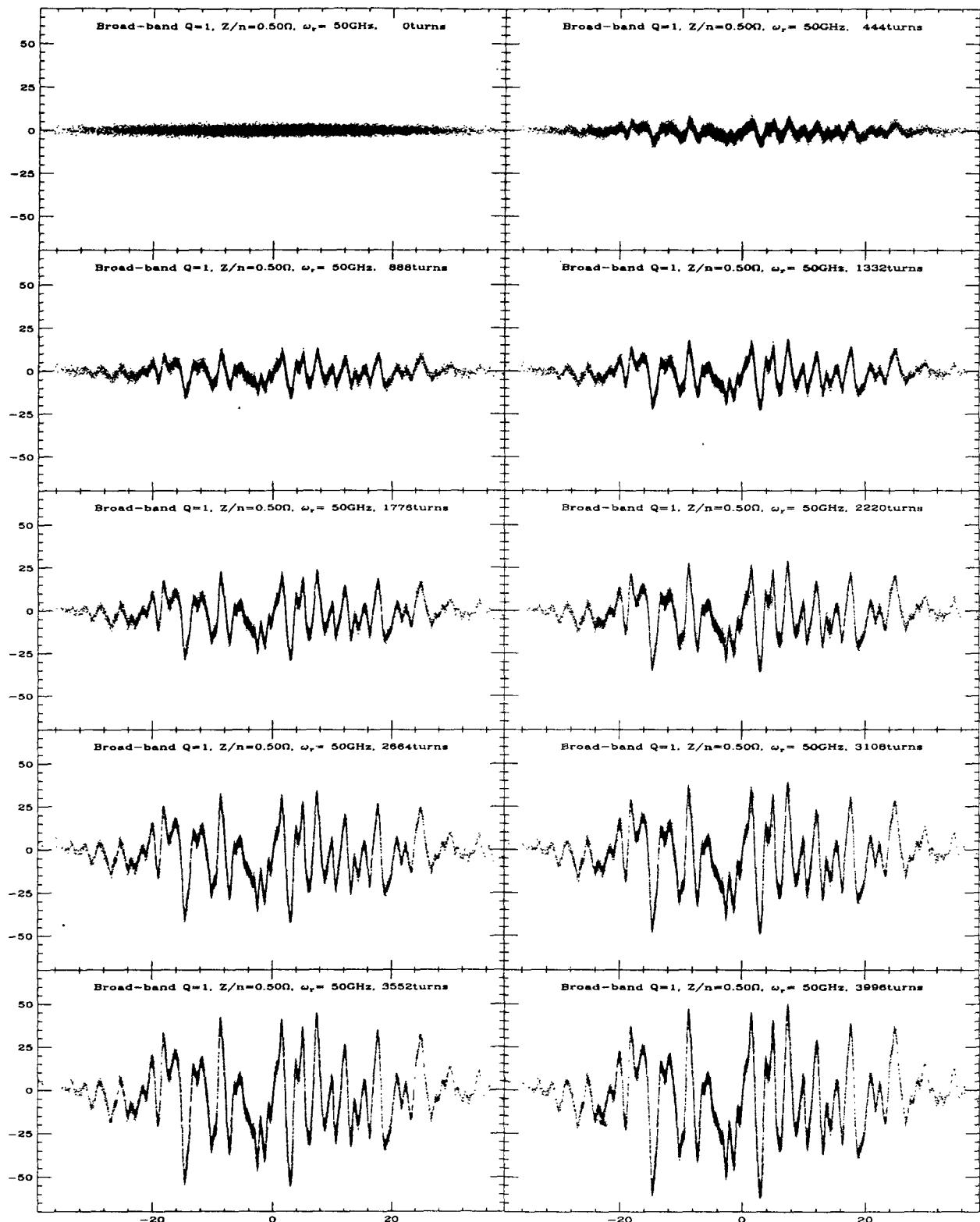
$$\gamma = -1 \times 10^{-6}$$



Energy offset in MeV vs distance in cm,  $E = 50$  GeV with  $\sigma_E = 1.5$  MeV,  $\sigma_t = 13$  cm.  $\eta = +1 \times 10^{-6}$ ,  $4 \times 10^{12}$  particles ( $2 \times 10^6$  macro), half-triangle width 15 ps. Compensating rf's: 65.341, 24.645 kV,  $\omega = 2421.83773$ , 5005.13131 MHz,  $\phi = 178.60214^\circ, 177.15095^\circ$ . Decay of muons has been included, 4000 turns, File: 50ghz-5-13-decay+eta-6-4000turns-2e6.

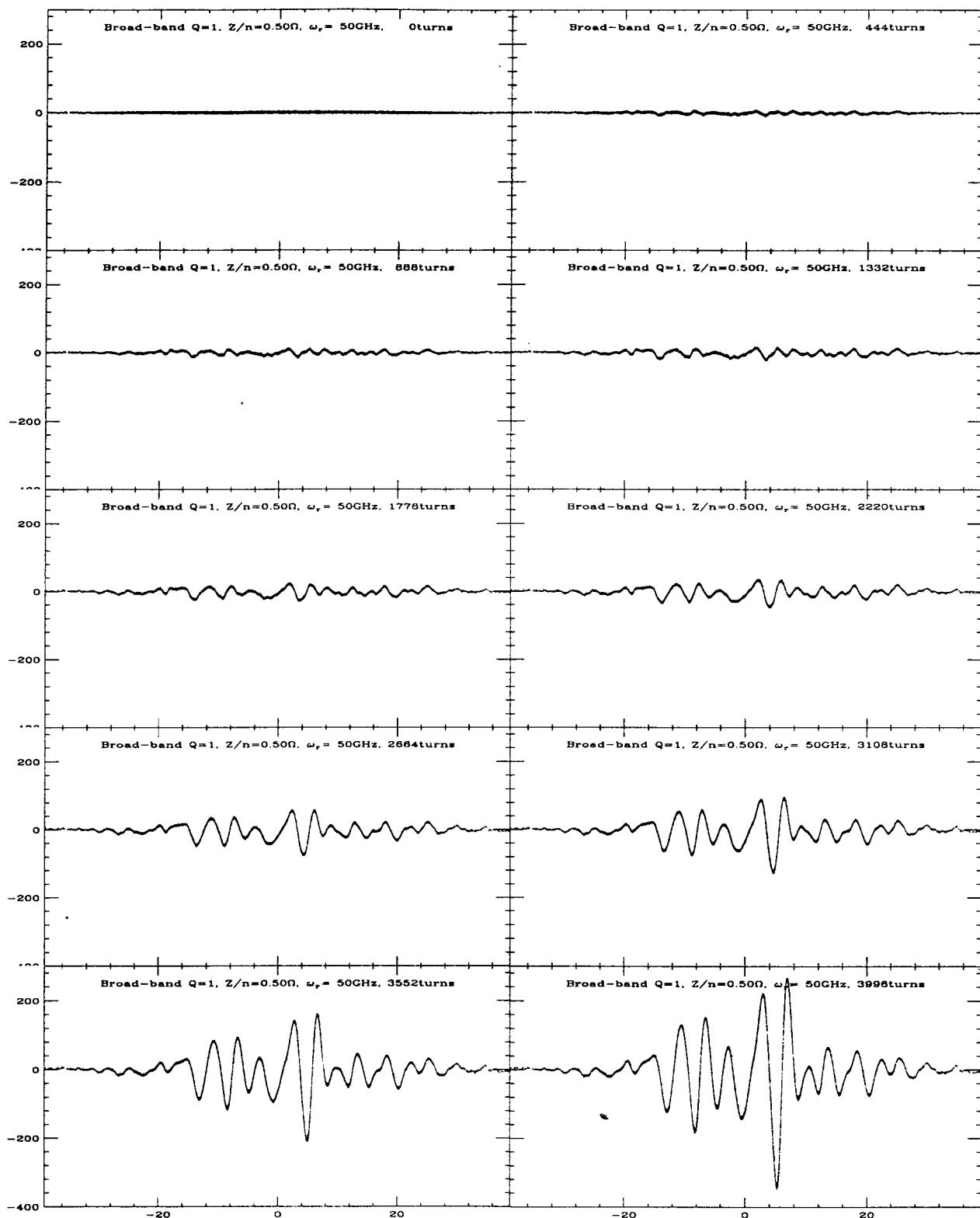
Shows wave growth

$$\eta = +1 \times 10^{-6}$$



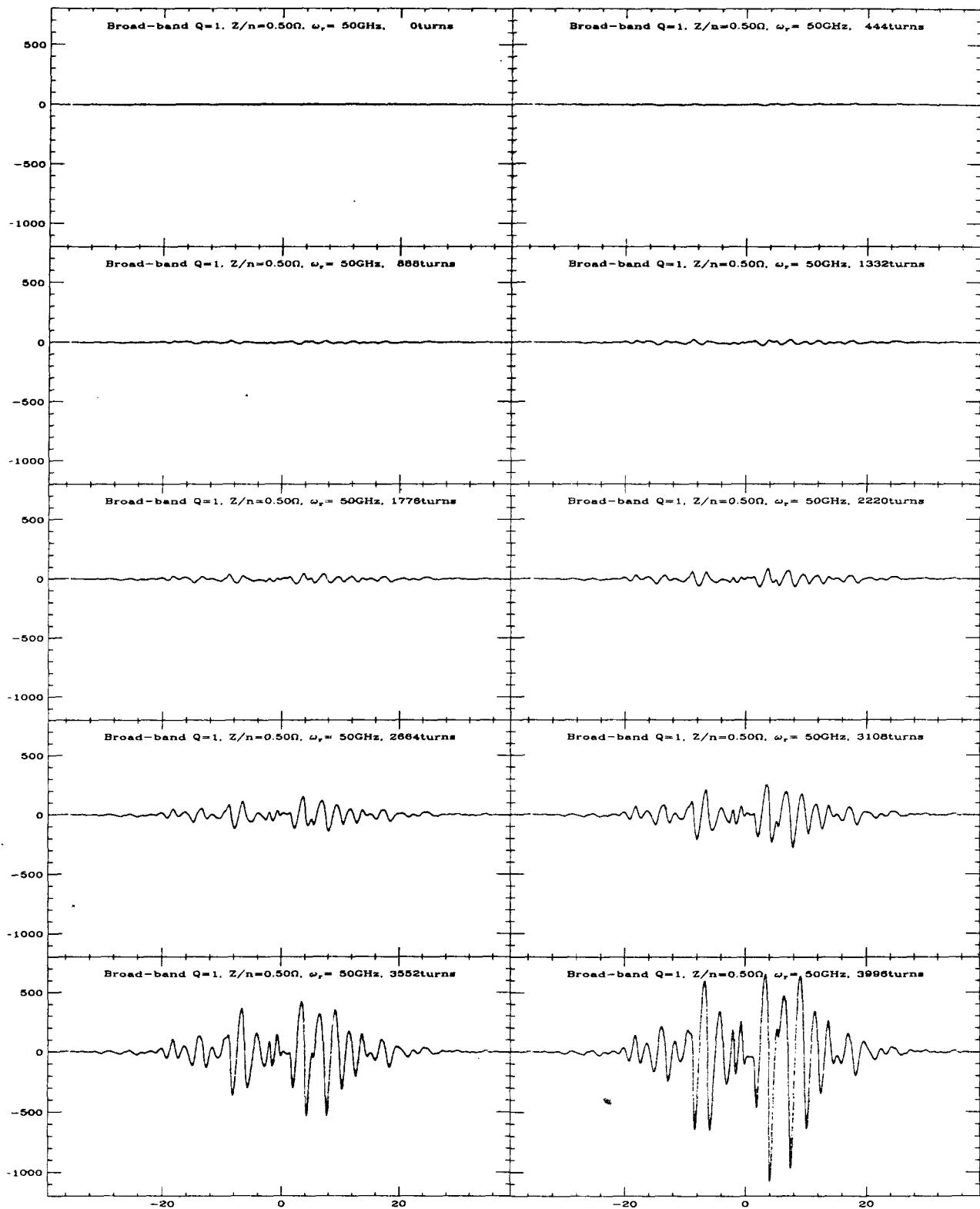
Energy offset in MeV vs distance in cm,  $E = 50 \text{ GeV}$  with  $\sigma_E = 1.5 \text{ MeV}$ ,  $\sigma_t = 13 \text{ cm}$ .  $\eta = -1 \times 10^{-14}$ ,  $4 \times 10^{12}$  particles ( $2 \times 10^6$  macro), half-triangle width 15 ps. Compensating rf's: 65.341, 24.645 kV,  $\omega = 2421.83773$ , 5005.13131 MHz,  $\phi = 178.60214^\circ, 177.15095^\circ$ . Decay of muons has NOT been included, 4000 turns, File: 50ghz-5-13-nodecay-eta-14-4000turns-2e6.

$\gamma \approx 1 \times 10^{-14}$  almost zero  
 $\therefore$  no p-wave effect



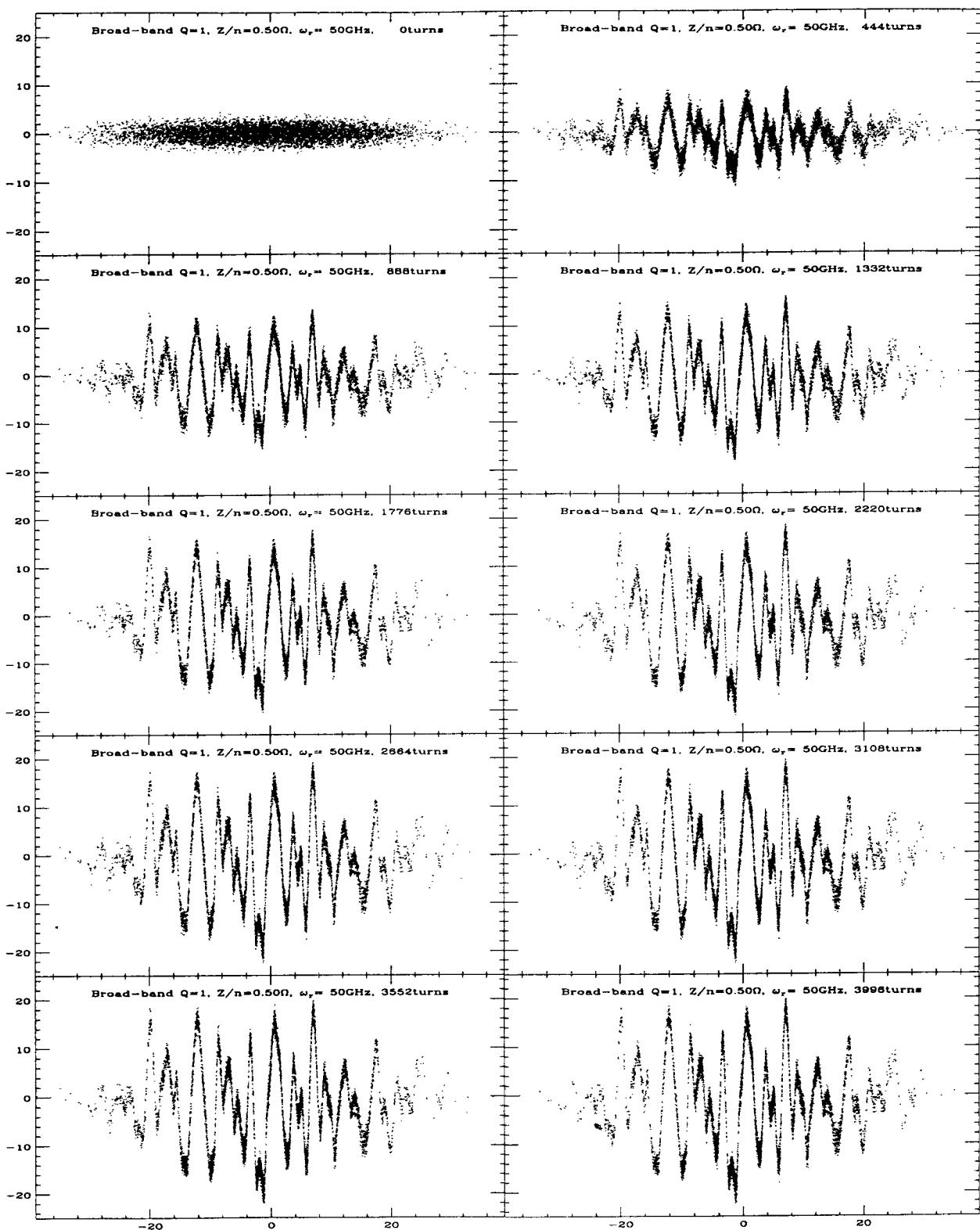
Energy offset in MeV vs distance in cm,  $E = 50 \text{ GeV}$  with  $\sigma_E = 1.5 \text{ MeV}$ ,  $\sigma_\ell = 13 \text{ cm}$ .  $\eta = -1 \times 10^{-6}$ ,  $4 \times 10^{12}$  particles ( $2 \times 10^6$  macro), half-triangle width 15 ps. Compensating rf's: 65.341, 24.645 kV,  $\omega = 2421.83773$ , 5005.13131 MHz,  $\phi = 178.60214^\circ, 177.15095^\circ$ . Decay of muons has NOT been included, 4000 turns, File: 50ghz-5-13-nodecay-eta-6-4000turns-2e6.

*below transition no decay of  $\mu$*

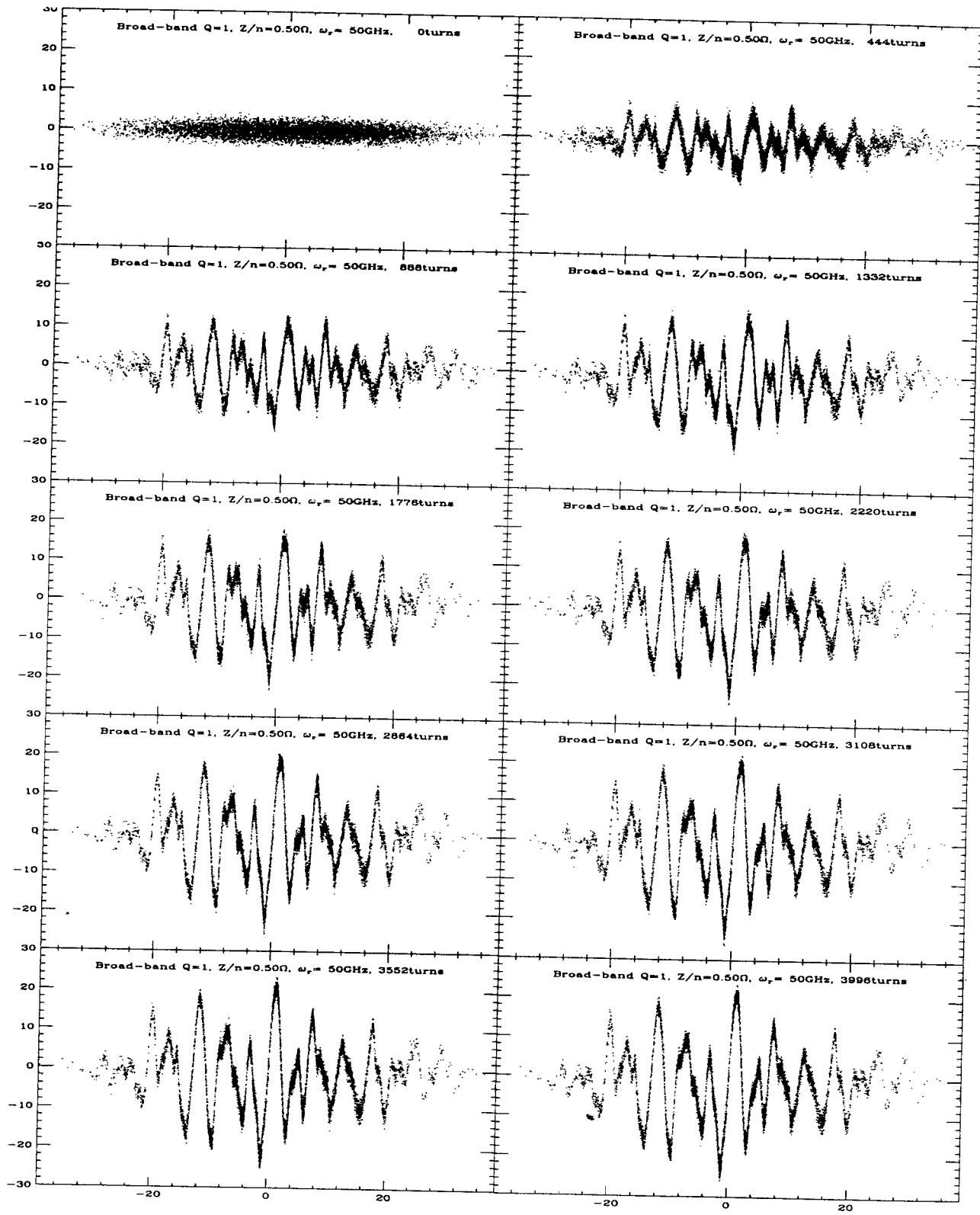


Energy offset in MeV vs distance in cm,  $E = 50 \text{ GeV}$  with  $\sigma_E = 1.5 \text{ MeV}$ ,  $\sigma_t = 13 \text{ cm}$ .  $\eta = +1 \times 10^{-6}$ ,  $4 \times 10^{12}$  particles ( $2 \times 10^6$  macro), half-triangle width 15 ps. Compensating rf's: 65.341, 24.645 kV,  $\omega = 2421.83773$ , 5005.13131 MHz,  $\phi = 178.60214^\circ, 177.15095^\circ$ . Decay of muons has NOT been included, 4000 turns, File: 50ghz-5-13-nodecay+eta-6-4000turns-2e6.

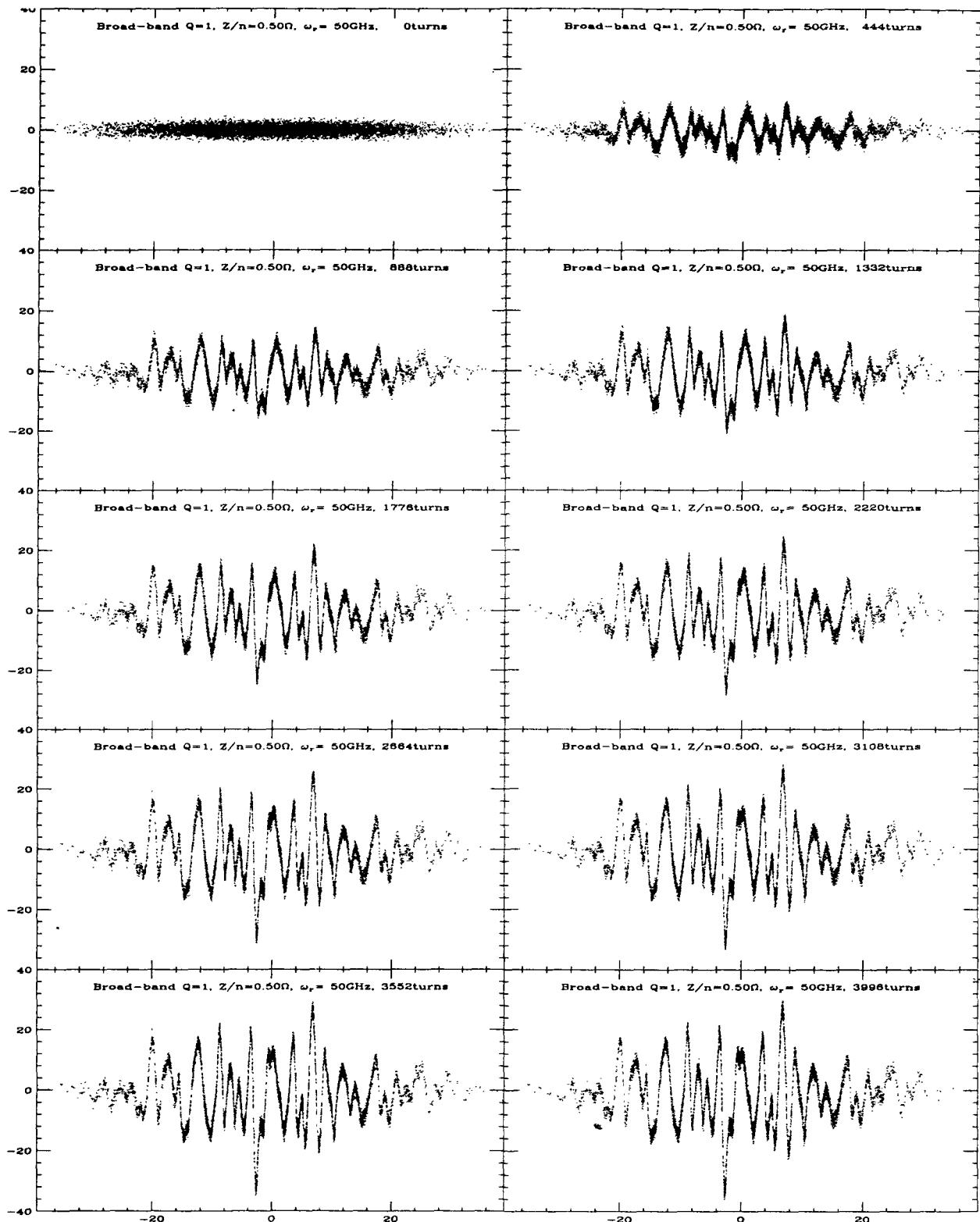
*above transition  
no decay of muons*



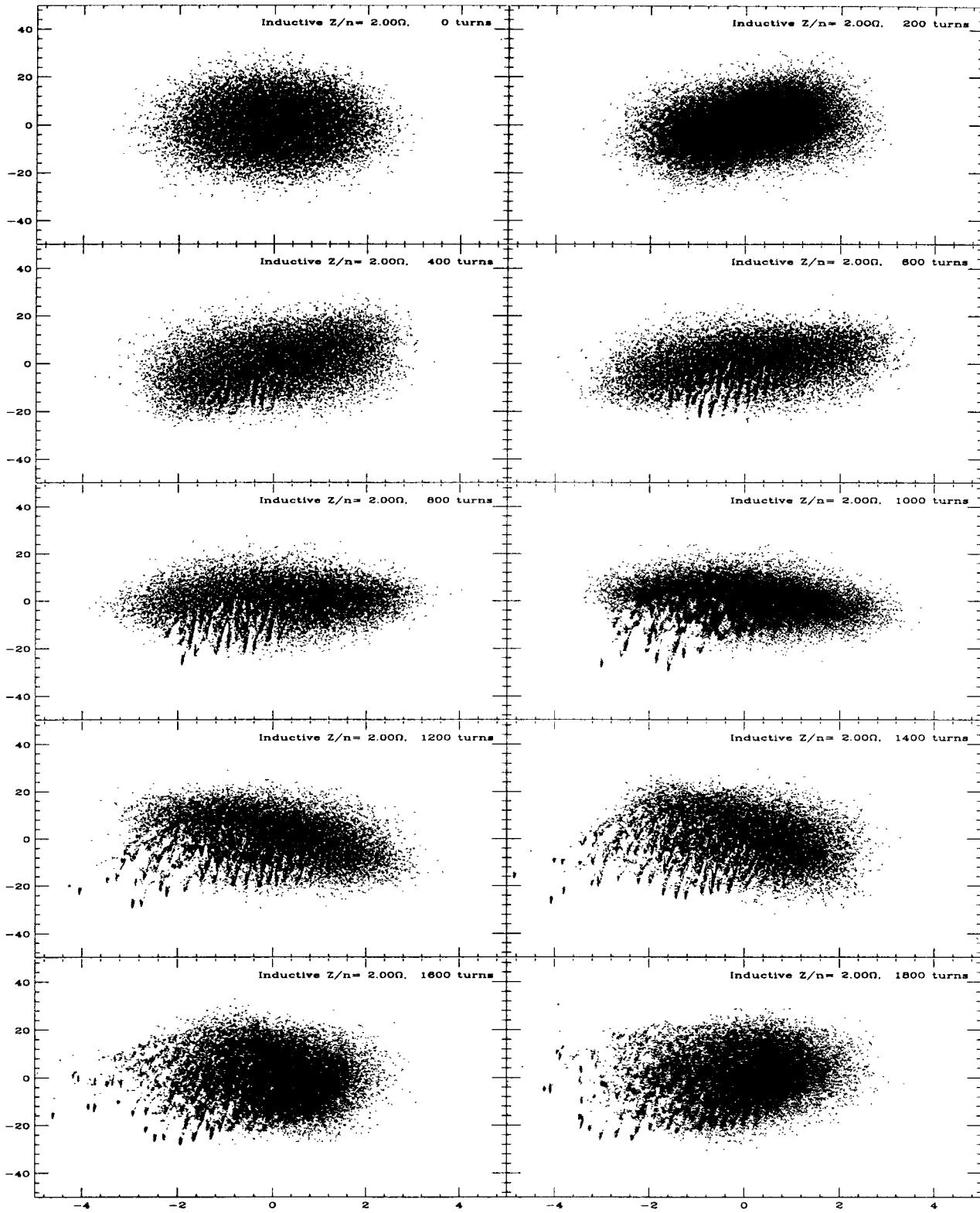
Energy offset in MeV vs distance in cm,  $E = 50$  GeV with  $\sigma_E = 1.5$  MeV,  $\sigma_\ell = 13$  cm.  $\eta = -1 \times 10^{-14}$ ,  $4 \times 10^{12}$  particles ( $1 \times 10^6$  macro), half-triangle width 15 ps. Compensating rf's: 65.341, 24.645 kV,  $\omega = 2421.83773$ , 5005.13131 MHz,  $\phi = 178.60214^\circ, 177.15095^\circ$ . Decay of muons has been included, 4000 turns, File: 50ghz-5-13-decay-eta-14-4000turns-1e6.



Energy offset in MeV vs distance in cm,  $E = 50$  GeV with  $\sigma_E = 1.5$  MeV,  $\sigma_t = 13$  cm.  $\eta = -1 \times 10^{-6}$ ,  $4 \times 10^{12}$  particles ( $1 \times 10^6$  macro), half-triangle width 15 ps. Compensating rf's: 65.341, 24.645 kV,  $\omega = 2421.83773$ , 5005.13131 MHz,  $\phi = 178.60214^\circ, 177.15095^\circ$ . Decay of muons has been included, 4000 turns, File: 50ghz-5-13-decay-eta-6-4000turns-1e6.

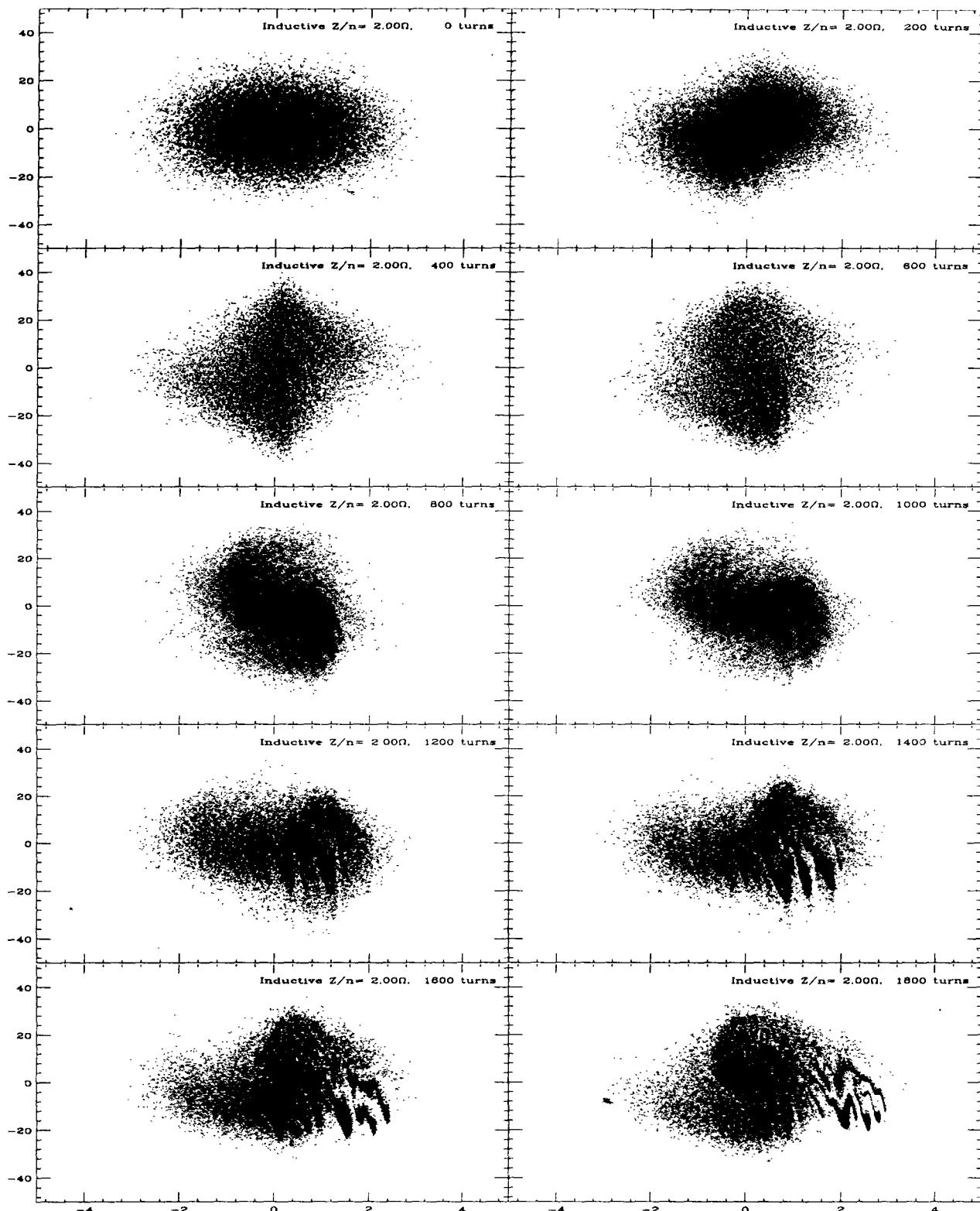


Energy offset in MeV vs distance in cm,  $E = 50$  GeV with  $\sigma_E = 1.5$  MeV,  $\sigma_t = 13$  cm.  $\eta = +1 \times 10^{-6}$ ,  $4 \times 10^{12}$  particles ( $1 \times 10^6$  macro), half-triangle width 15 ps. Compensating rf's: 65.341, 24.645 kV,  $\omega = 2421.83773$ , 5005.13131 MHz,  $\phi = 178.60214^\circ, 177.15095^\circ$ . Decay of muons has been included, File: 50ghz-5-13-decay+eta-6-4000turns-1e6.



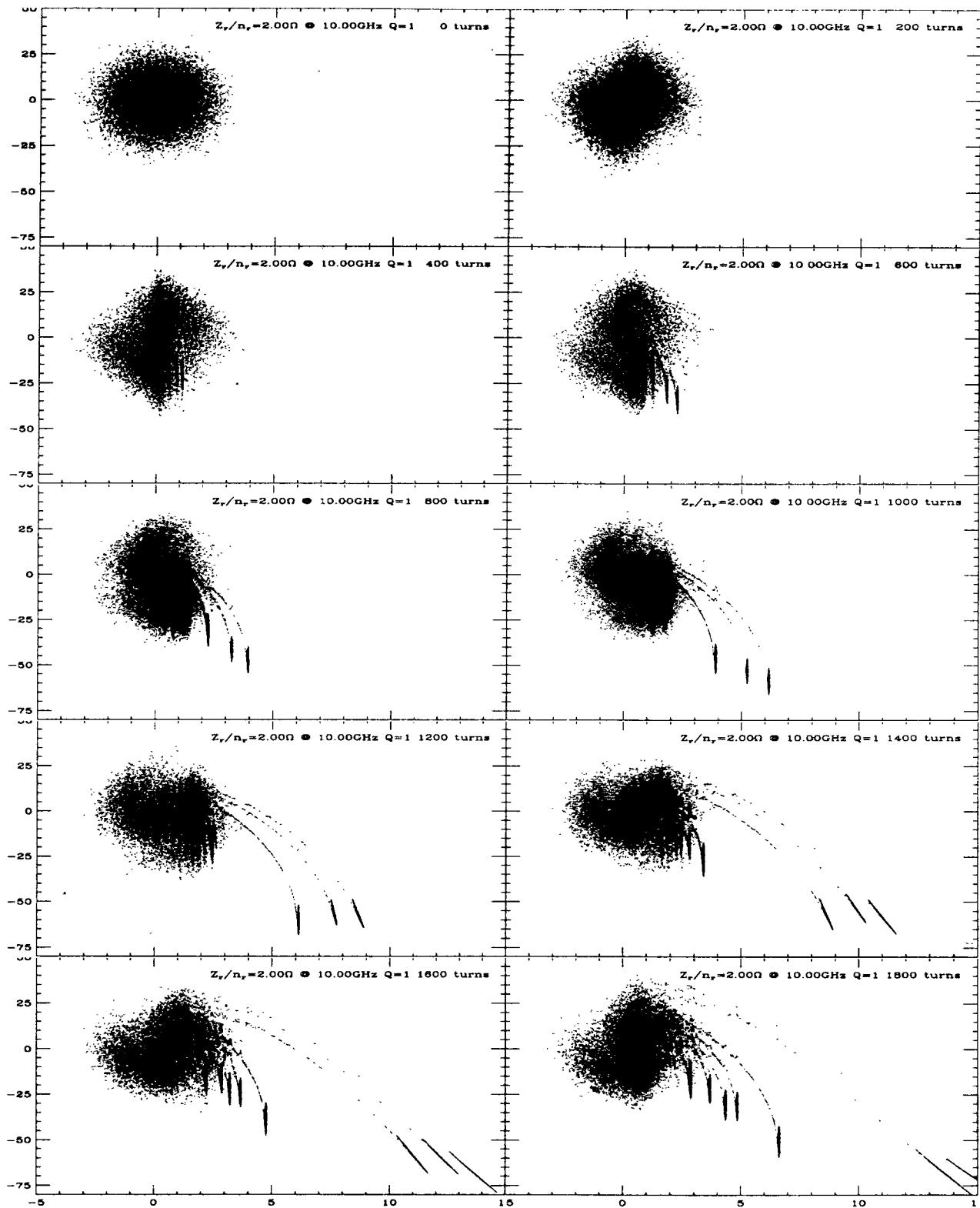
Energy offset in MeV vs time in ns,  $E = 100$  GeV with rms Gaussian spread 0.0001,  $\eta = +0.001, 9.83013 \times 10^{10}$  particles (400000 macro), half-triangle width 0.05 ns, frequency-independent pure reactive wake with inductive  $Z/n = 2.00 \Omega$ .

*above transition  
pure inductive, 2 times above threshold*



Energy offset in MeV vs time in ns,  $E = 100$  GeV with rms Gaussian spread 0.0001,  $\eta = -0.001$ ,  $9.83013 \times 10^{10}$  particles (400000 macro), half-triangle width 0.05 ns, frequency-independent pure reactive wake with inductive  $Z/n = 2.00 \Omega$ .

*below transition  
pure inductive, 2 x times above threshold*



Energy offset in MeV vs time in ns,  $E = 100 \text{ GeV}$  with rms Gaussian spread  $0.0001$ ,  $\eta = -0.001$ ,  $9.83013 \times 10^{10}$  particles (400000 macro), half-triangle width  $0.05 \text{ ns}$ , broad-band impedance with  $Z_r/n_r = 2.00 \Omega$  at  $10.0 \text{ GHz}$ .

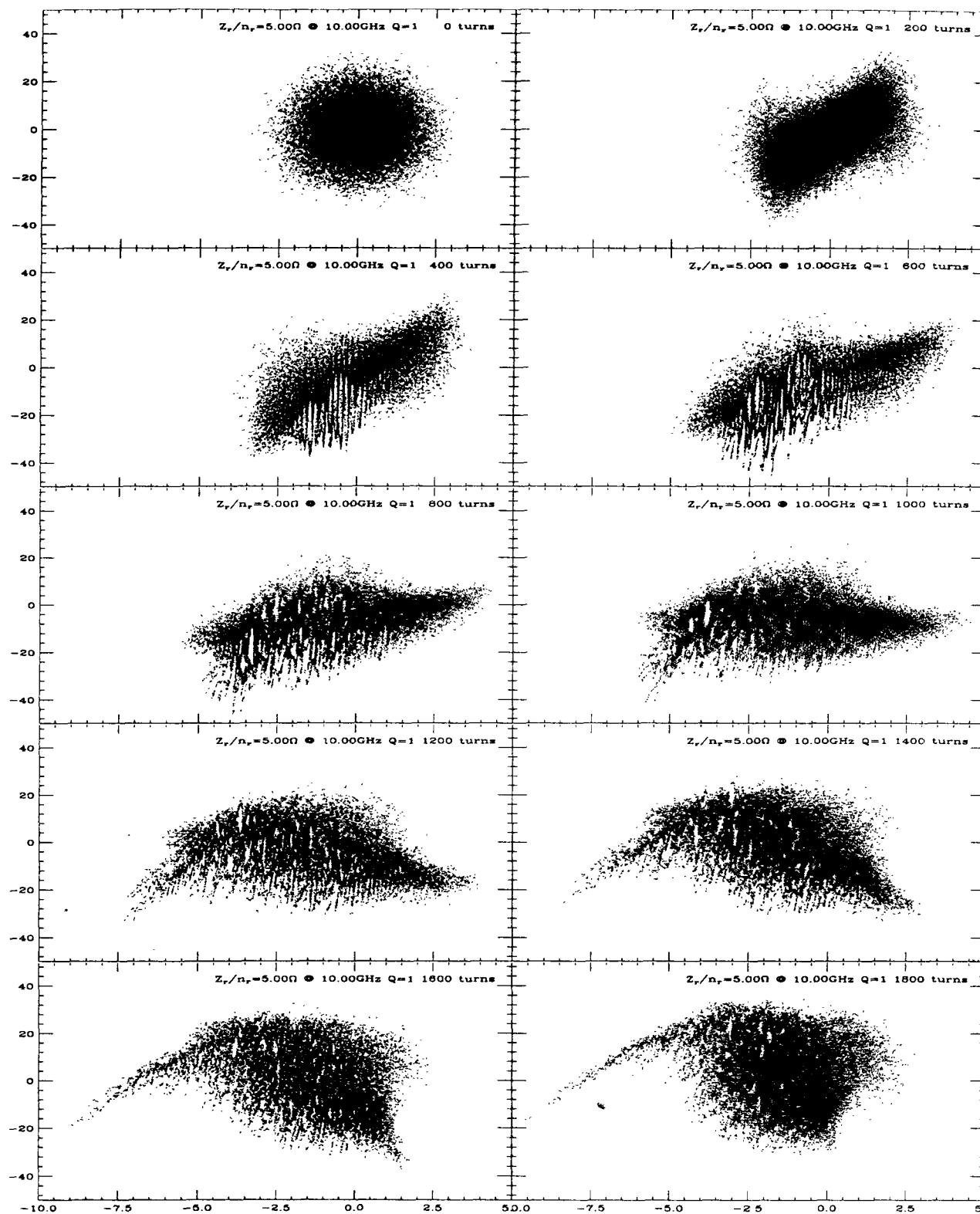
*below transition*

broad band  $Q=1$

$$\frac{z}{n}$$

2 times above threshold

$$v_{\text{max}} = b_0^{\frac{1}{2}} = 2 \text{ cm} \cdot \frac{c}{\gamma} = 10$$

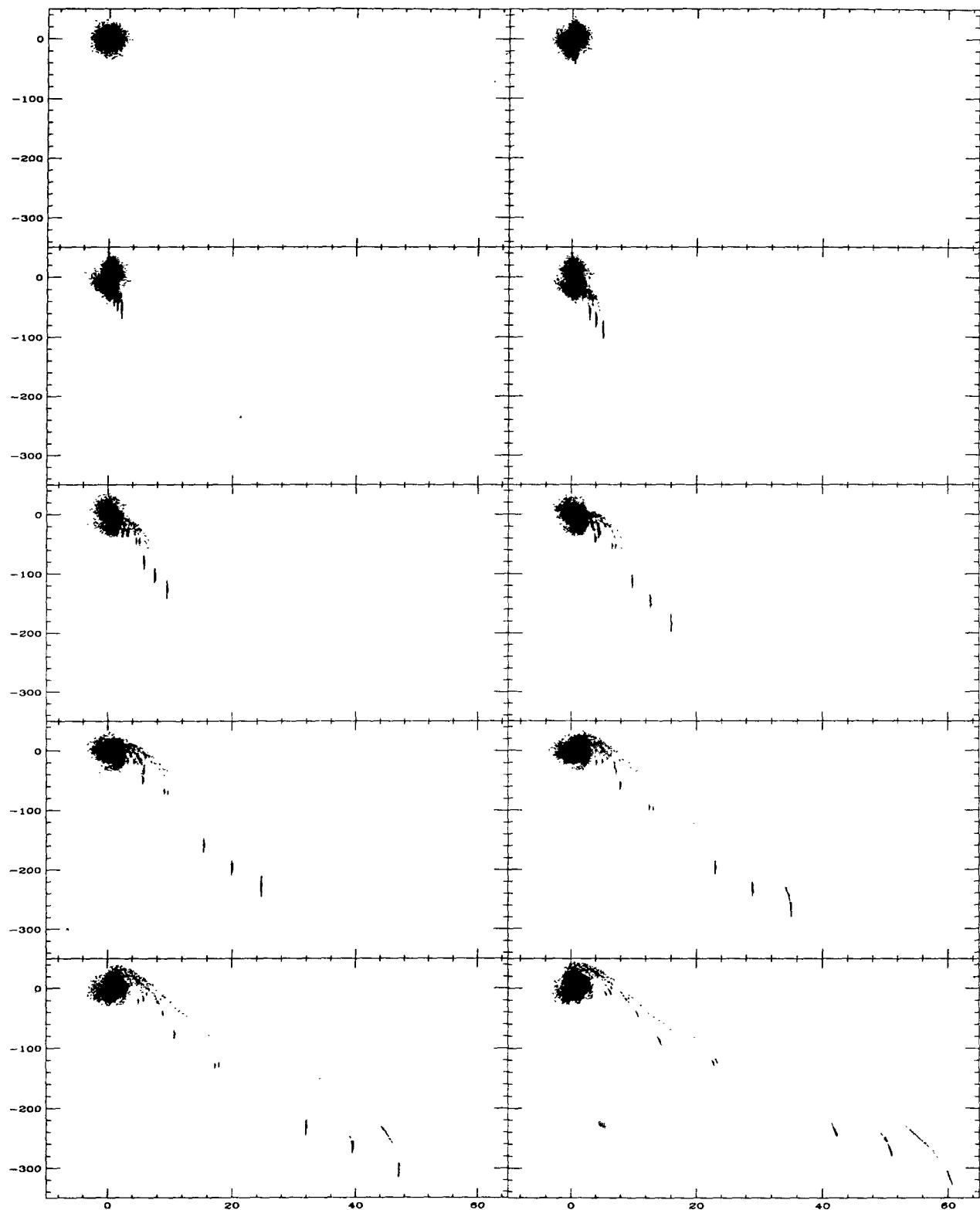


Energy offset in MeV vs time in ns,  $E = 100$  GeV with rms Gaussian spread 0.0001,  $\eta = +0.001, 9.83013 \times 10^{10}$  particles (400000 macro), half-triangle width 0.05 ns, broad-band impedance with  $Z_r/n_r = 5.00 \Omega$  at 10.0 GHz.

Above transition

broad band imp  $Q=1$   $\frac{Z}{n}$  5 times above threshold

thb-5.00-5-10

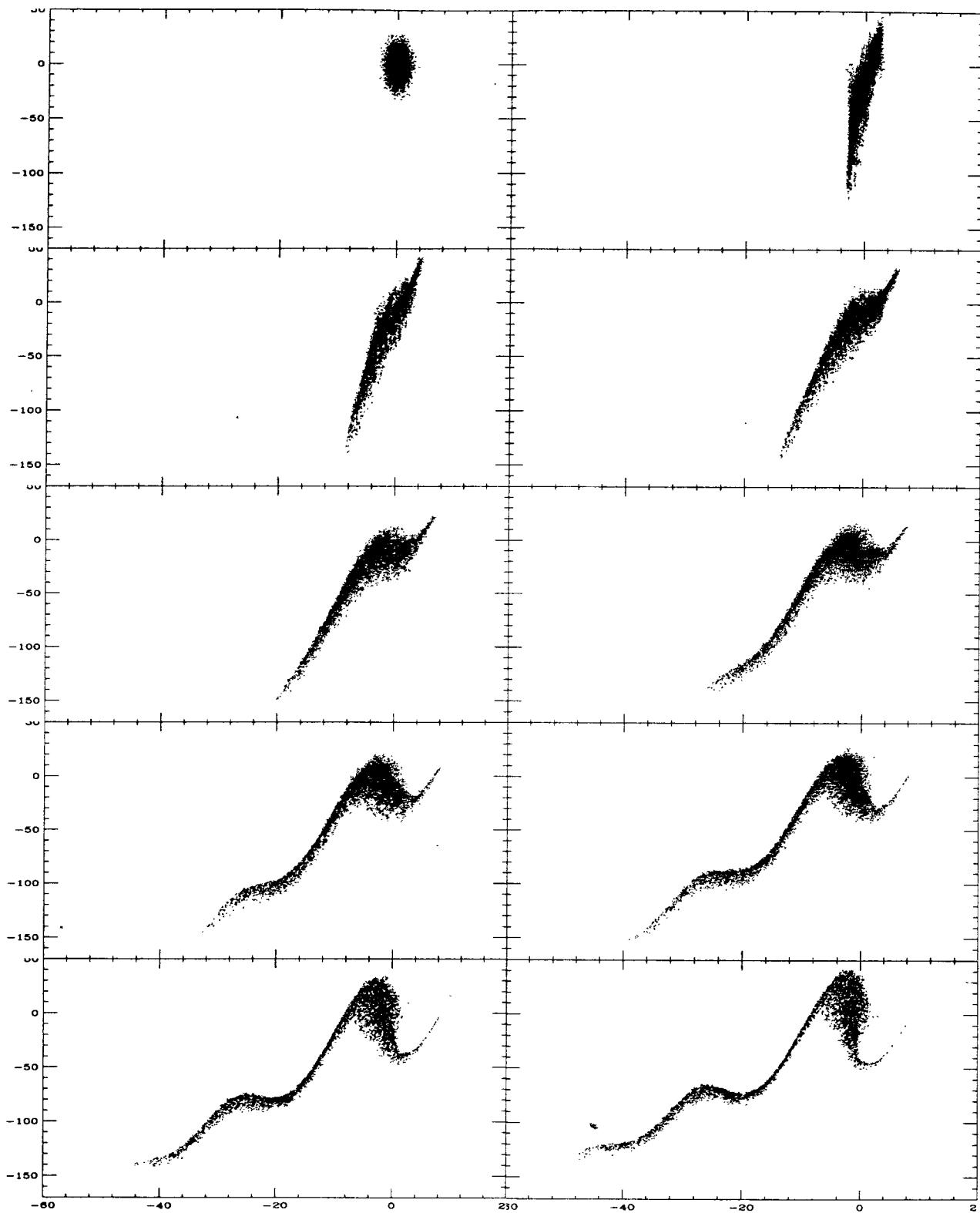


Energy offset in MeV vs time in ns,  $E = 100$  GeV with rms Gaussian spread 0.0001,  $\eta = -0.001$ ,  $9.83013 \times 10^{10}$  particles (400000 macro), half-triangle width 0.05 ns, broad band of  $Z/n = 8.0 \Omega$ ,  $Q = 3$  at 10 GHz. File: micro-bb-8,00-,5-10-q3,0-new

Below transition

Broad band imp.  $Q=3$

$\frac{Z}{n}$  8 times above threshold



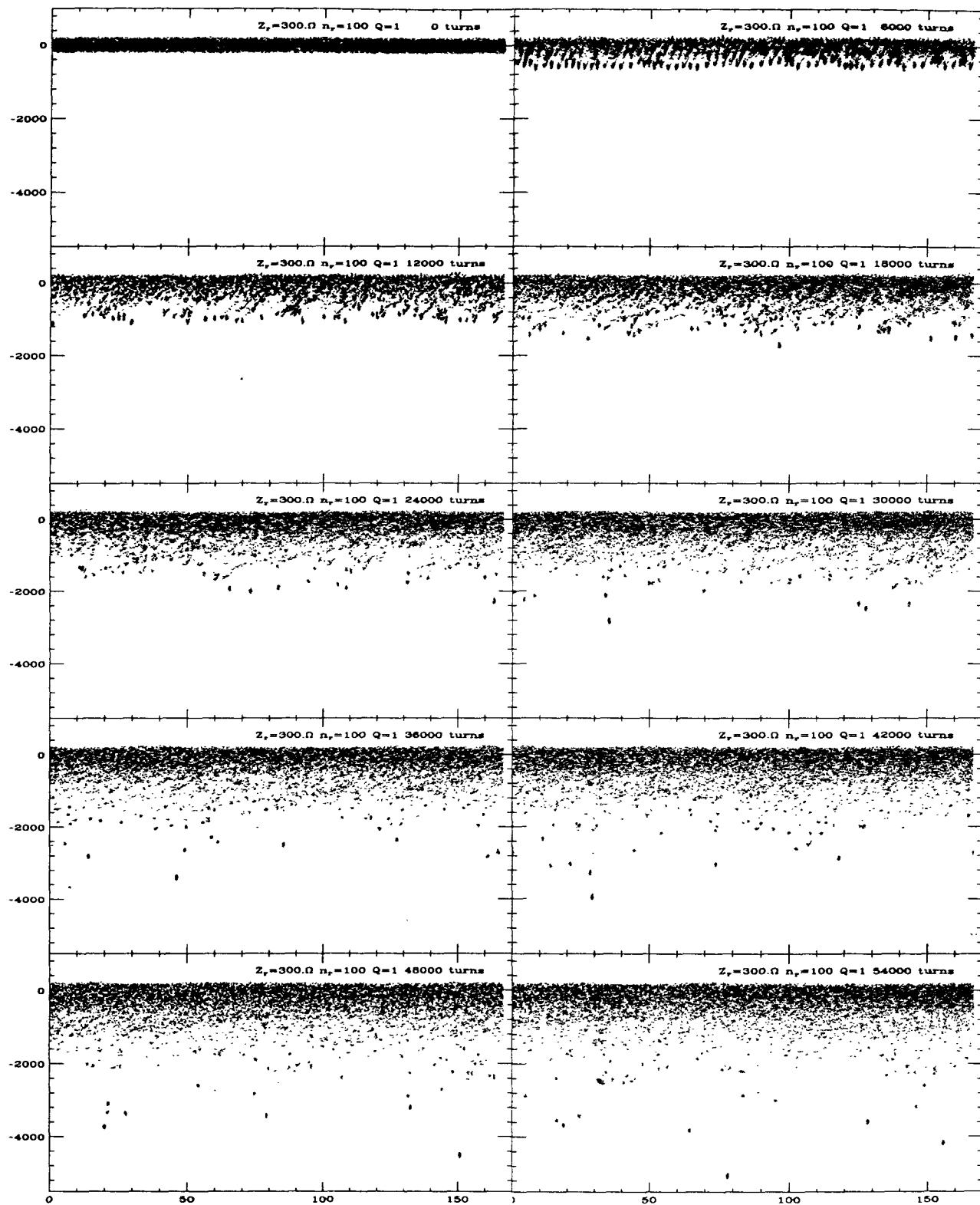
Energy offset in MeV vs time in ns,  $E = 100$  GeV with rms Gaussian spread 0.0001,  $\eta = +0.001$ ,  $9.83013 \times 10^{10}$  particles (400000 macro), half-triangle width 0.05 ns, broad band of  $Z/n = 16.0 \Omega$ ,  $Q = 1$  at 10 GHz. File: micro+bb-16,00-,5-10-q1,0-new

Above Transition

broad-band imp.  $Q=1$

$$\frac{Z}{n}$$

16 times above threshold



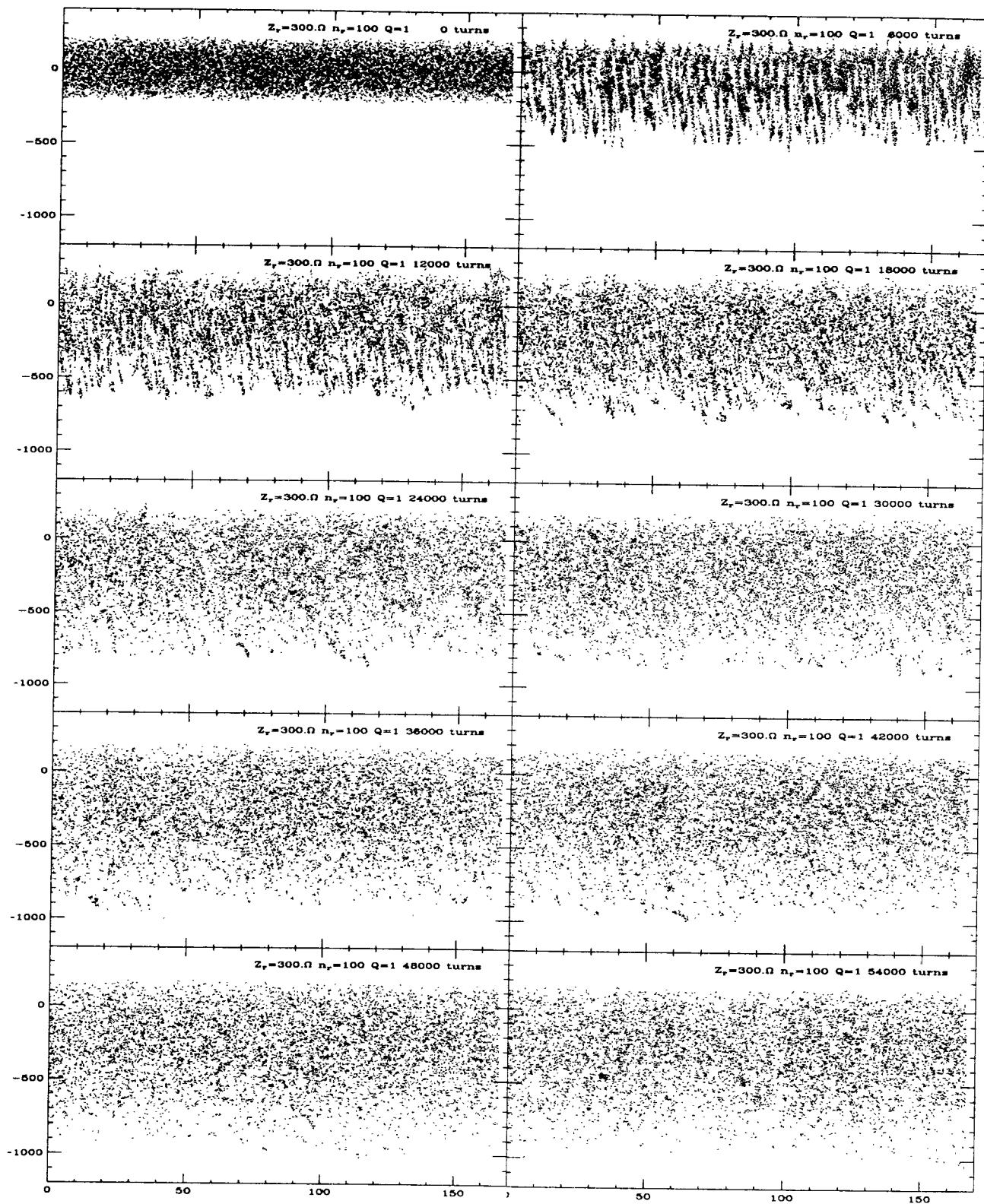
Energy offset in MeV vs time in ns. Ring circum. 50 m,  $E = 100$  GeV with rms parabolic spread 0.001,  $\eta = +0.005$ ,  $3.27 \times 10^{15}$  particles (800000 macro), 400 bins, resonant impedance  $Z/n = 3.0 \Omega$  (limit is  $1.0 \Omega$ ).

File: coast-para-300-100+400

initial  $\sigma_E = 100$  MeV

above transition

broad-band imp.  $Q=1$   $\frac{Z}{n} = 3$  times threshold



Energy offset in MeV vs time in ns. Ring circum. 50 m,  $E = 100$  GeV with rms parabolic spread 0.001,  $\eta = -0.005$ ,  $3.27 \times 10^{15}$  particles (800000 macro), 400 bins, resonant impedance  $Z/n = 3.0 \Omega$  (limit is  $1.0 \Omega$ ).  
File: coast-para-300-100-400

initial  $\sigma_E = 100$  MeV

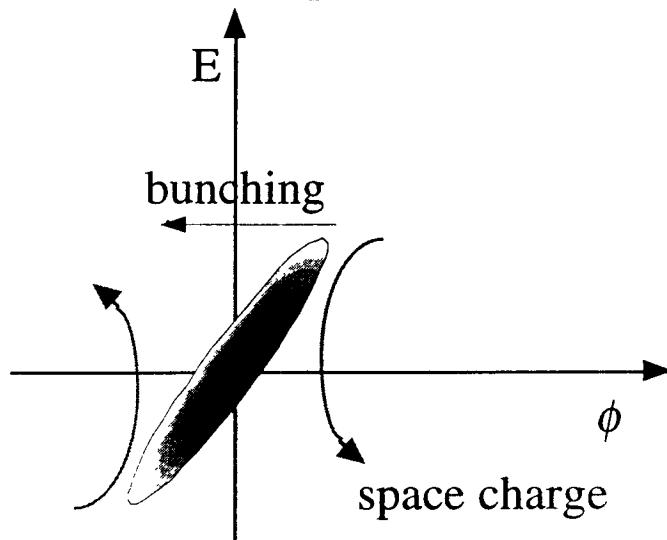
below transition

broad band imp.  $Q=1$

$\frac{Z}{n} = 3$  times threshold

## A possible experiment at KEK . . .

- We have demonstrated that bunching is possible in the absence of space charge. Space charge can, however, prevent bunching due to self fields which rotate the bunch before it can be compressed.



- A possible solution to this problem is to move the transition energy far from the beam energy to make the final bunch rotation fast enough so that the space charge effects do not contribute. This requires  $| \gamma - \gamma_t | \sim 2$ .
- This can be tested at KEK where the  $\gamma_t$  jump waveform is a half sine wave. (The bunch charge will be low.)

